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Shearing interferometry for wavefront sensing

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Zerinke polynomials and beyond

CONCLUSION

Shearing interferometry is a large framework,

in which most of the wavefront sensors industrially developped

(Hartmann, Shack-Hartmann, lateral shearing interferometers, multi-lateral shearing interferometers, deflectometry, …)

can be included

allowing a theoretical common description and so,

a fair evaluation, in the different contexts of application

Tentative agenda

Irradiance Transport Equation (ITE)

Curvature Sensor

« Variations on a Hartmann theme »

Geometric versus Interferometric

Wave-front reconstruction from derivatives,

Wave-front sensing : perspectives

How to define Wavefront sensing ?

Here : analysis of the phase of a wave coming from a point (notice that the point is actually present or not)

typically :

- limited to surfaces (can be generalized)
- amplitudes $\lambda/10$ to 100 λ
- precision less than $\lambda/100$ to $\lambda/4$ rms
- spatial variations large with respect to amplitudes (mm / µm)
- often related to real time operation
- often, related to a correction set-up

Nowadays, there is no clear definition of wavefront sensing

The purpose of this talk is to propose a common framework

Main applications:

Optical testing:

Lenses (phones, camera, telescopes, …)

Active or adaptive optics:

Astronomy (atmospheric turbulence)

Intense lasers (focal spot shaping)

Ophthalmology (test, retina observation)

Image restauration:

Astronomy (atmospheric turbulence)

Irradiance Transport Equation (ITE)

Curvature Sensor

« Variations on a Hartmann theme »

pause

Geometric versus Interferometric

Wave-front reconstruction from derivatives,

Wave-front sensing : perspectives

Irradiance Transport Equation

Wave-front sensors : a lot of set-ups

All are based on a same principle:

The wave-front is deduced from
the observation of a propagated irradiance profile

Irradiance Transport Equation

(M. Teague, J. Opt. Soc.Am. A, Vol2, N°11, Nov 1985)

Irradiance Transport Equation (1)

Wave A(x) is composed of an irradiance I and a phase Φ

$$A(x,y)=\sqrt{I(x,y)}\,.e^{i\Phi(x,y)}$$

We want to know Φ,

However, sensors are sensitive to I

Basic Idea :

Measuring the variations of I

during its propagation

(variations essentially related to Φ)

In order to obtain Φ

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Irradiance Transport Equation (2)

$$A_z(x,y) = e^{ikz} A_o(x,y) * \frac{e^{i\pi\frac{(x^2+y^2)}{\lambda z}}}{i\lambda z}$$

Propagation in the context of parabolic approximation,

Fresnel operator,

A, solution of :

$$\left(i\frac{d}{dz} + \frac{\nabla^2}{2k} + k \right) A_z(x,y) = 0 \qquad k = \frac{2\pi}{\lambda}$$

Taking into account that I = A.A*, we obtain:

$$-\frac{2\pi}{\lambda}\frac{\partial I}{\partial z} = I\nabla^2\Phi + \nabla I.\nabla\Phi$$

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Irradiance Transport Equation (3)

∇ Nabla operator : a two dim vector, x-derivative, and y-derivative

Nabla²: scalar product of nabla

$$\nabla = \begin{bmatrix} \frac{d}{dx} \\ \frac{d}{dy} \end{bmatrix}$$

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

$$-\frac{2\pi}{\lambda}\frac{\partial I}{\partial z} = I\left(\frac{d^2\Phi}{dx^2} + \frac{d^2\Phi}{dy^2}\right) + \frac{dI}{dx}\frac{d\Phi}{dx} + \frac{dI}{dy}\frac{d\Phi}{dy}$$

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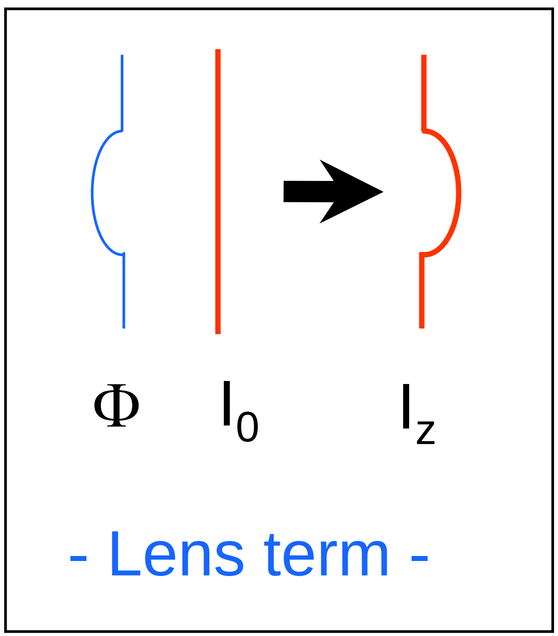
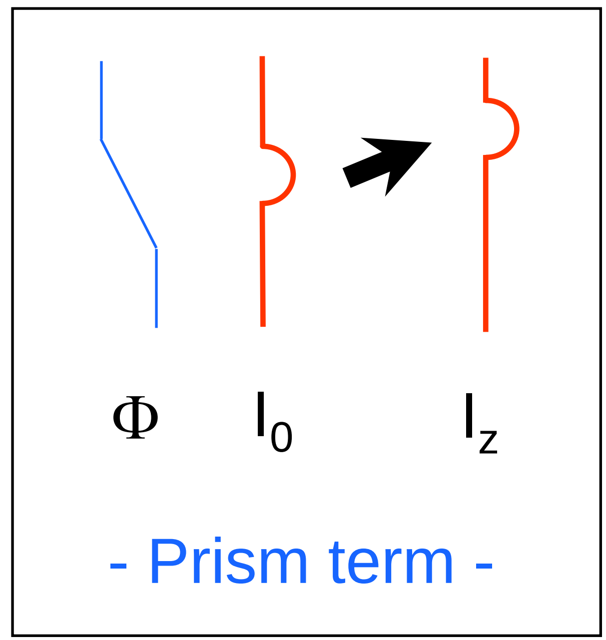
Irradiance Transport Equation, physical meaning

$$I_z(x,y) = I_0(x,y) - \frac{\lambda z}{2\pi}\left(I_0 \nabla^2 \Phi_0 + \nabla I_0 . \nabla \Phi_0\right)$$





Basic principle of wavefront sensing

***First strategy:* I observe irradiance variations due to local curvatures of the wave-front, during propagation.**

« lens » effect

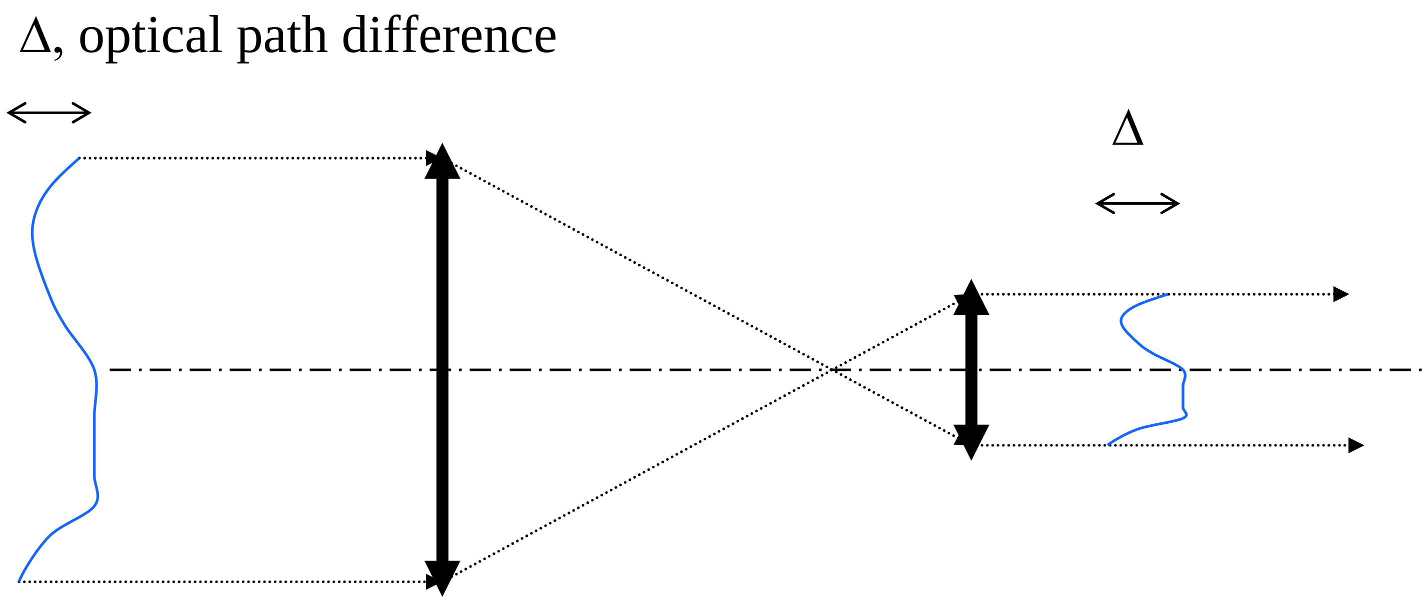
(“ombroscopie”, curvature sensor**)**

***Second strategy:* I insert a periodic profile in the beam to be analyzed and I observe the local distortions of this profile during its progation. Distortions are related to local tilts in the wave-front.**

« prism » effect

(Hartmann and variations, lateral shearing interferometry, deflectometry…)

Gouy theorem, sensitivity and compactness



As Δ is conservative during the propagation (Gouy),
reducing the pupil is a good way to enhance the sensitivity
1/magnification for tilts
1/magnification^2 for curvatures

Example : In astronomy, 1m to 10 m entrance pupil, 1 cm exit pupil !

Irradiance Transport Equation (5)

Achromaticity : a general property for this family

Optical path difference instead of phase in the equation

Φ ➔ Δ, with Φ = $2\pi\,\Delta / \lambda$

$$-\frac{\partial I}{\partial z} = I\nabla^2\Delta + \nabla I . \nabla\Delta$$

if I, I modulated and the OPD Δ are independent of λ,

➔ then the variation of I versus z is independent of λ,

➔ and a measurement in white light conditions is possible

Irradiance Transport Equation (6)

$$-\frac{\partial I}{\partial z} = I\nabla^2\Delta + \nabla I.\nabla\Delta$$

A Warning for « prism » family:

To preserve the achromatism, you need an achromatic way to introduce the periodic profile in the analyzed beam (for example, a simple grid)

For example, a shearing plate is not achromatic (and all the systems related to Michelson interferometer)

Achromatism interest

Δ independent of λ :

- mirror defects
- atmospheric turbulence

achromatism allows a better signal-to-noise ratio

Femto Laser : pulses are chromatic,

measurement is only possible with an achromatic set-up

Irradiance Transport Equation (ITE)

Curvature Sensor

« Variations on a Hartmann theme »

Geometric versus Interferometric

Wave-front reconstruction from derivatives,

Wave-front sensing : perspectives

Curvature sensor : lens effect
an historical example, the Chinese magic mirror

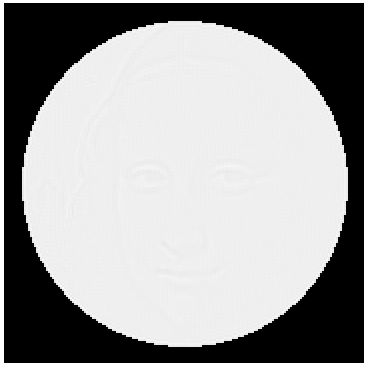
The chinese magic mirror is a metallic mirror, slightly hammered, to obtain a pattern in relief, with an amplitude of nearly one micron.

Black levels are converted in elevation

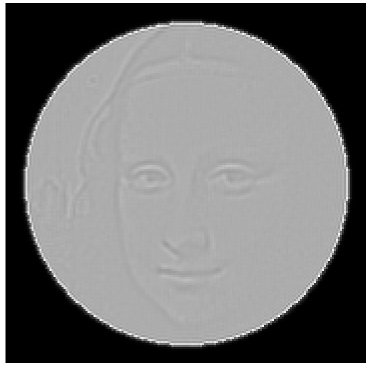
Phi

Miroir

Magic Mirror, reflecting a source at infinity,
the reflected irradiance is observed on a wall at different distances



Visu



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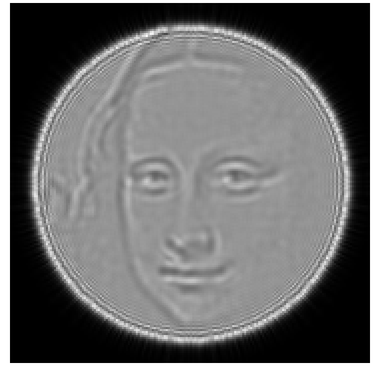

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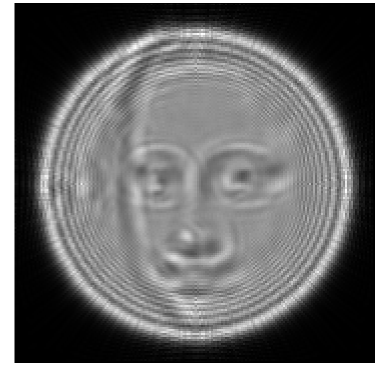

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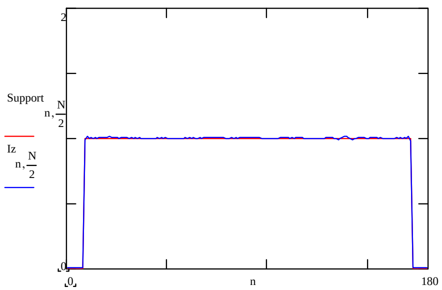

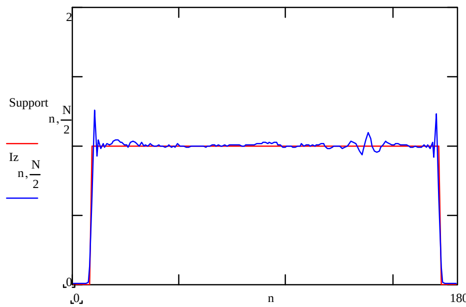

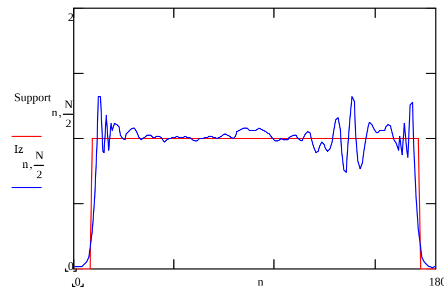

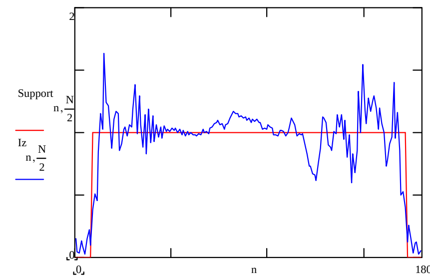

Reconstruction from the curvature maps (integrate twice)

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Curvature sensor: Main points

- **Sensitivity is directly proportional to the distance of the observation plane**
- **Spatial resolution is inversely proportional to the distance of the observation plane**

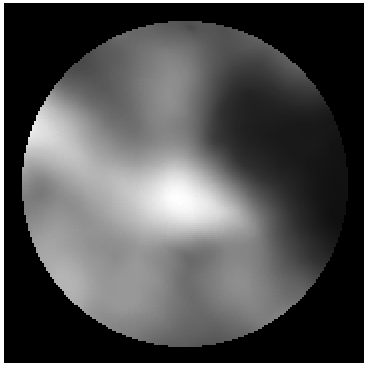
A necessary trade-off for the user :

- **Irradiance variations have to be as large as possible, to have a good detection of curvatures (good signal-to-noise conditions)**
- **=> *you have to maximize the distance***
- **On the other hand, spatial resolution has to be preserved**
- **=> *you have to minimize the distance***

This necessary trade-off is true for all the set-ups described here !

Problem : the natural scintillation in the beam (1)

Basic principle :to do a measurement in two planes, on both sides of the pupil
« positive » curvature ➔irradiance goes up
« negative » curvature ➔ irradiance goes down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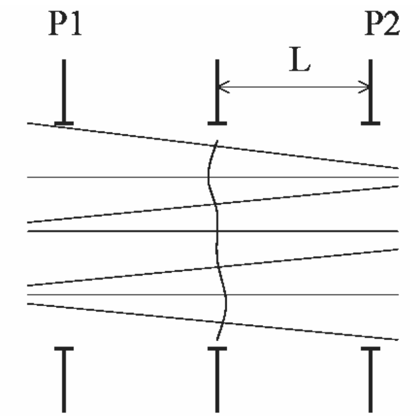

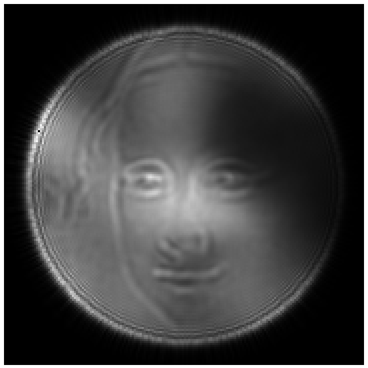
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Problem : the natural scintillation in the beam (2)

Be careful ! This is only an approximate way to compensate
You have to verify the relevance for each case
for example, for a gaussian beam (irradiance),
the method exposed here is no more relevant

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The reconstruction quality is directly affected by scintillation
even with a compensation

Irradiance Transport Equation (ITE)

Curvature Sensor

« Variations on a Hartmann theme »

Geometric versus Interferometric

Wave-front reconstruction from derivatives,

Wave-front sensing : perspectives

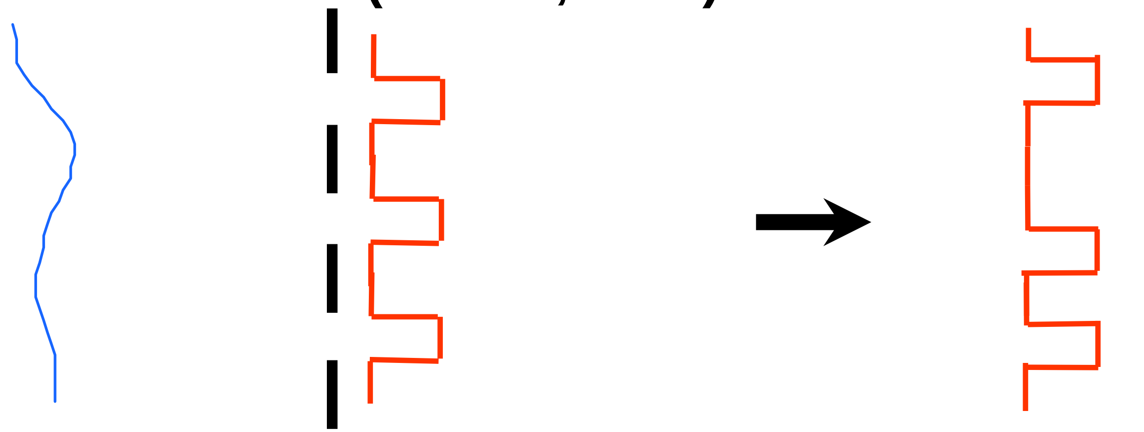
Variations on a Hartmann theme : prism effect

Principle :

The basic principle is to introduce a periodical modulation in the analyzed beam.

The simplest way to proceed is to insert a perforated mask in the plane in which analysis has to be done

Proposed by Hartmann, 1900, it is always of common use ➔ for example the Zelentchouk giant telescope has been verified by a Hartmann screen (Zverev, 1976)



Analyzed wavefront

Analysis plane
Hartmann screen
Cardboard mask for zelentchouk

Measurement plane
Photographic plate

« prism » versus « lens », how to do the good choice ?

Very similar to AM / FM / PM for telecom

Curvature sensor is based on a local measurement of irradiance.

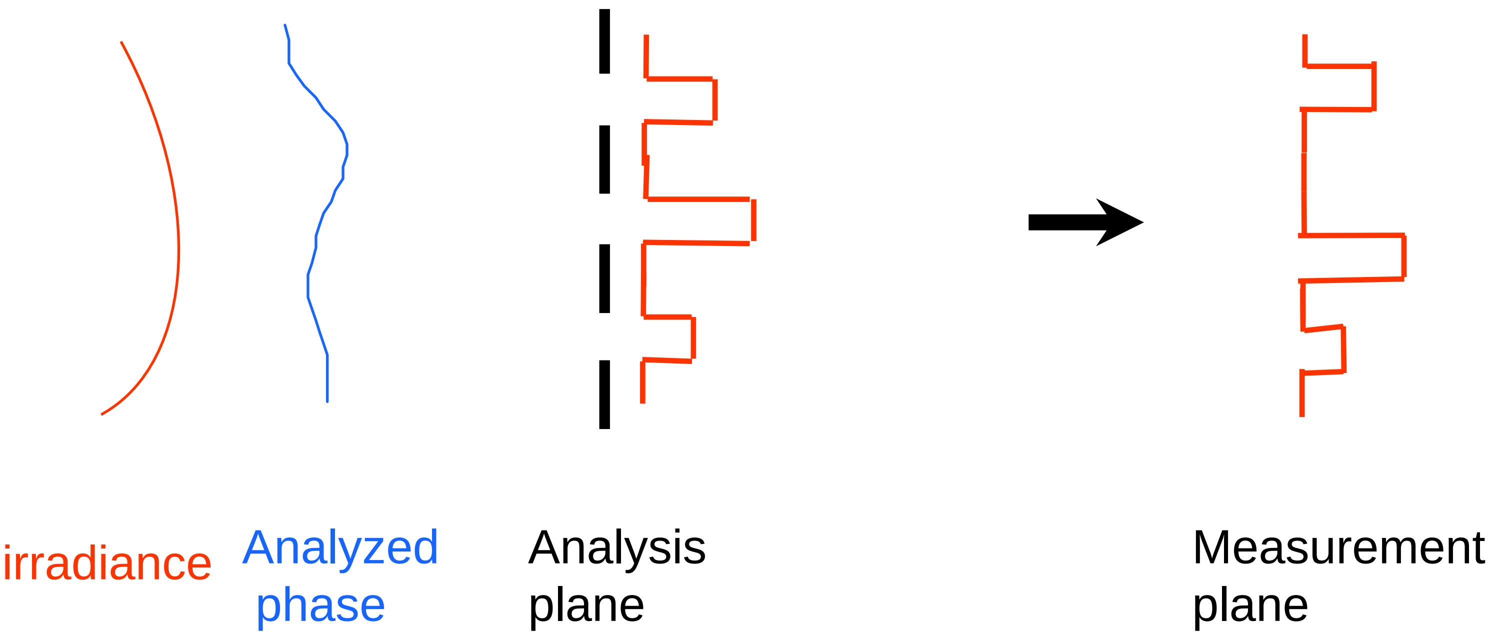
So its fundamental simplicity is counterbalanced by scintillation problems

At least, the measurement has to be done in two planes

Set-ups based on « prism » effect are looking for the distortions of a career frequency.

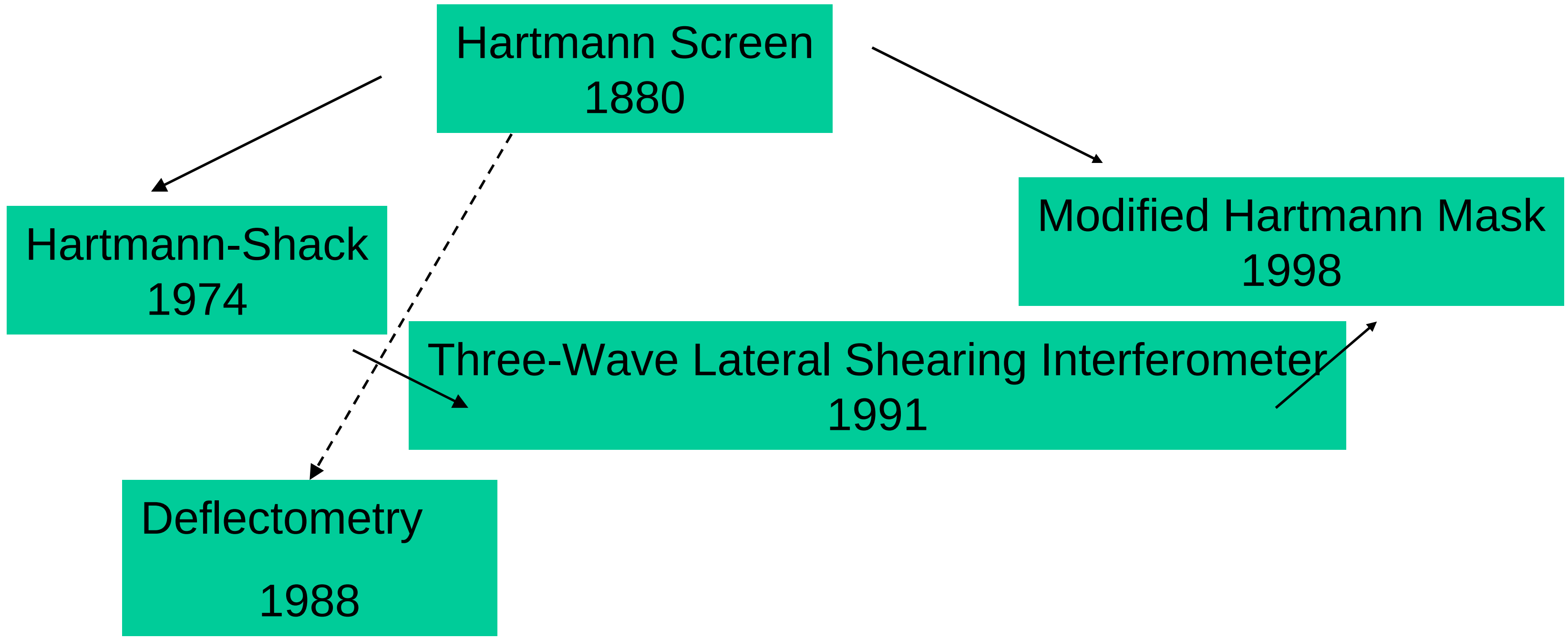
The complexity induced by the insertion of a periodical profile in the analyzed beam is counterbalanced by a global insensitivity to irradiance variations of this beam.

Irradiance variations in « prism » set-ups



Sensitivity can be adjusted, by translation of the measurement plane

Hartmann family members (1)



A rather complicated family tree

The good way to compare them :

They are all based on a same principle

« introduce an irradiance career frequency in order to measure the local gradients of the phase »

They differ by the chosen technical solution to introduce the career frequency

Hartmann family members (2)

You do not have an ideal wave-front sensor

The good choice depends on

- *Low or High Light level conditions*
- *Need of compactness*
- *Environment conditions (vibrations, …)*
- *Achromaticity (short pulses, …)*
- *Real time operation (adaptive optics, …)*
- *Metrology, control, control / command, …*

And other industrial constraints …

The right choice for a wave-front sensor is not strictly related to intrinsic performance of the reconstructed wave-front

Hartmann family members (3)

Warning !

Each of the members has been developed in a specific context (astronomy, metrology, optical testing, intense laser, …)

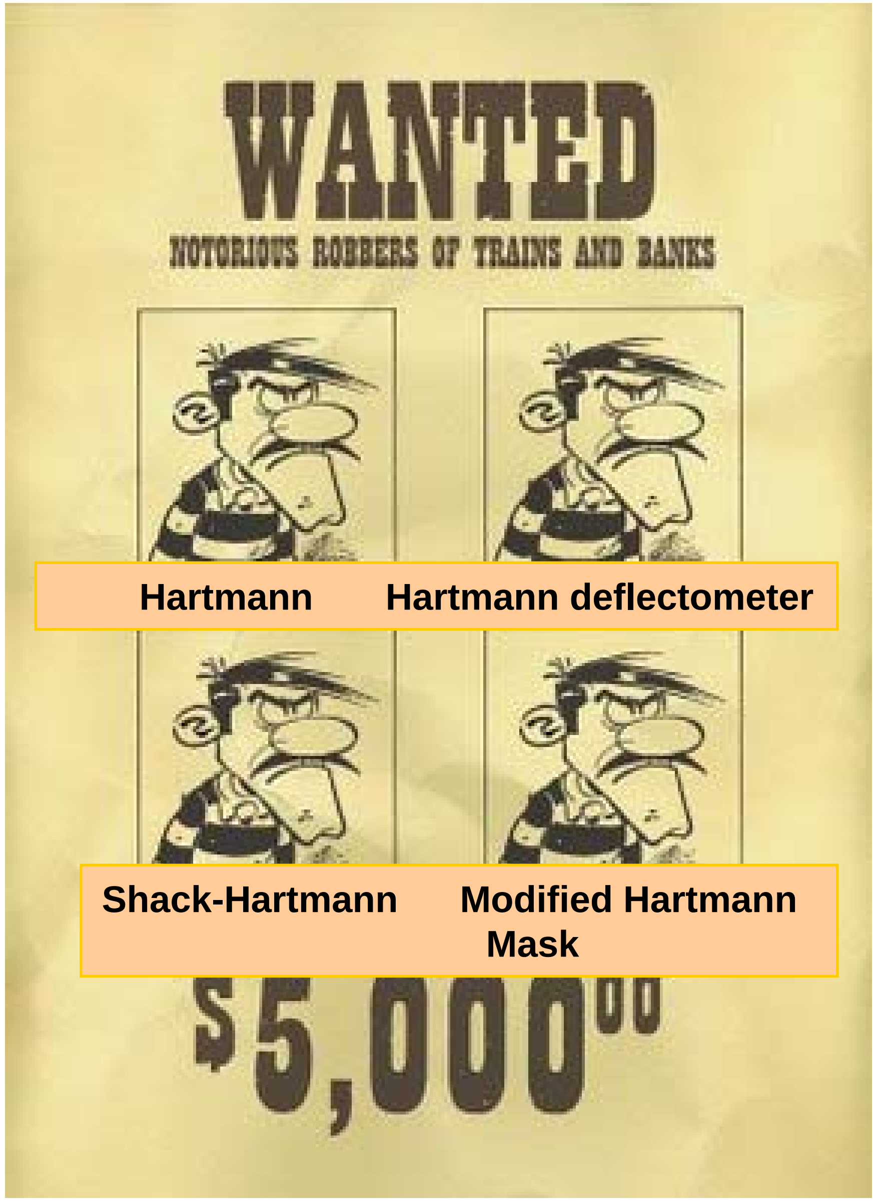
You have to be careful if you want to do a comparison between the set-ups.

You have to distinguish between intrinsic performances of the set-up and reached performances due to technical choices

Trite example :a wave-front sensor using a very sensitive detector has better performances in low-light level conditions than a standard wave-front sensor.

It is always interesting to take a little time to understand the bases of the wave-front sensors; it allows a prediction of the improvements you can reach for a specific modification

Hartmann cousins



Hartmann-Shack wave-front sensor (1)

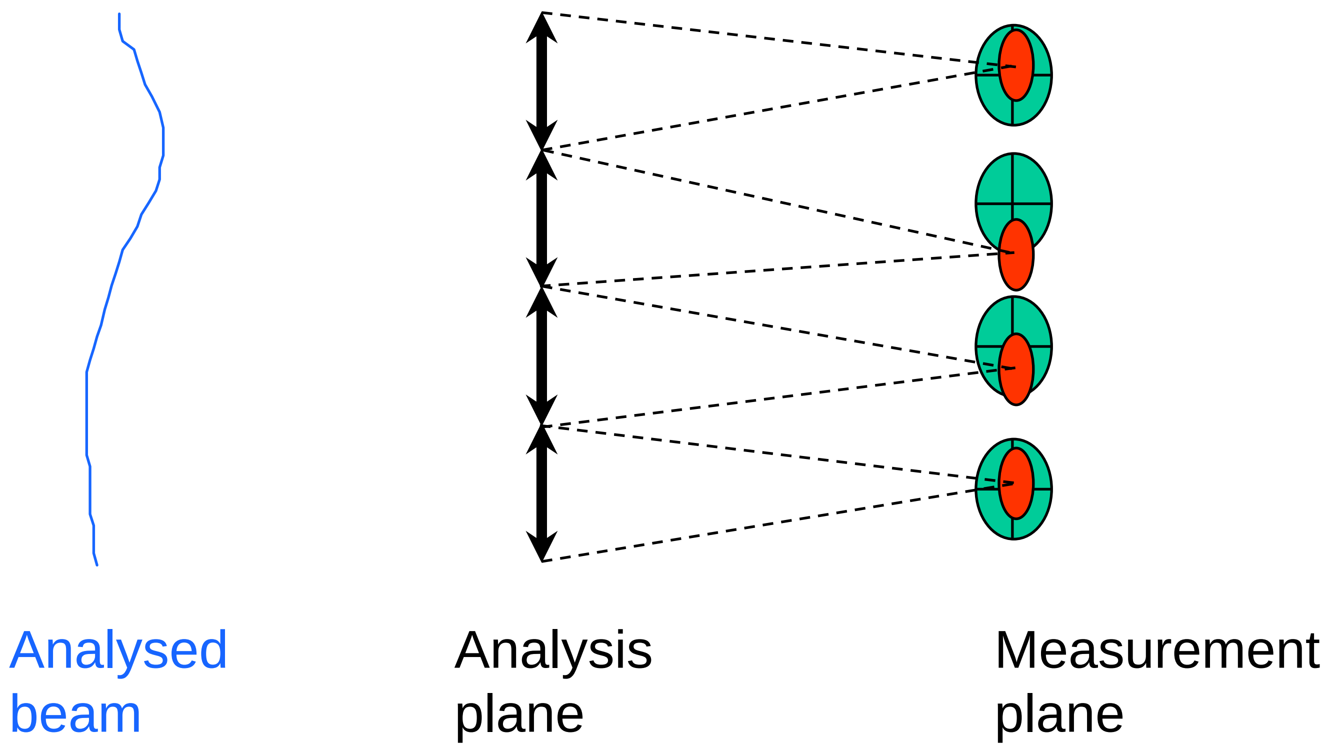
A discreet birth, as it is only referenced in a proceeding of a conference (J. Opt. Soc. Am. A, 1974)

The creation of HS has been lately related by Platt and Shack, the co-inventors of the set-up (Journal of Refractive Surgery, 2001)

At the beginning, a work done for US Air Force, for the real-time evaluation of the perturbations introduced by atmosphere, to restore the images of satellites observed from the Earth

Shack : a very fine specialist in optics (pentaprism, …) has the idea to renovate the Hartmann screen (classically used by his astronomer colleague) to avoid the global waste of energy (screen transmission is largely lower than 50%). As the technology in detection is not very sensitive, at that time, the preservation of impinging photons is the main problem for adaptive optics control.

Hartmann-Shack wave-front sensor (2)



Hartmann-Shack, the breakthrough (4)

Irradiance modulation is no more introduced in the analysis plane, but "coded" by a grid of microlenses, to appear in the measurement plane

This fundamental property could justify the creation of a new branch in interferometry

But, from the origin onwards, Shack considers this set-up as a simple variation of Hartmann screen (and do not publish)

In fact, HS has at least one specificity (not very used) which is not in Hartmann screen : you have real images of the source in the common focal plane of the microlenses

And one property is lost : sensitivity is no more continuously adjustable; the only possible observation plane is the common focal plane of the microlenses

Hartmann-Shack, a success story (5)

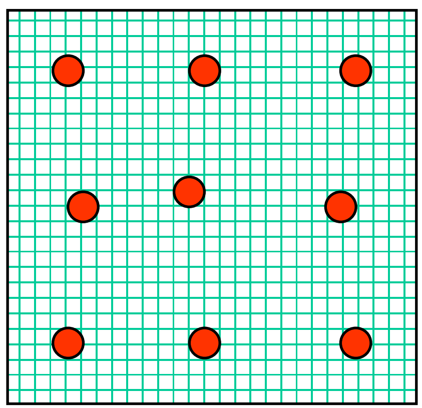
The success of a set-up is in the encounter between

a need

and an idea

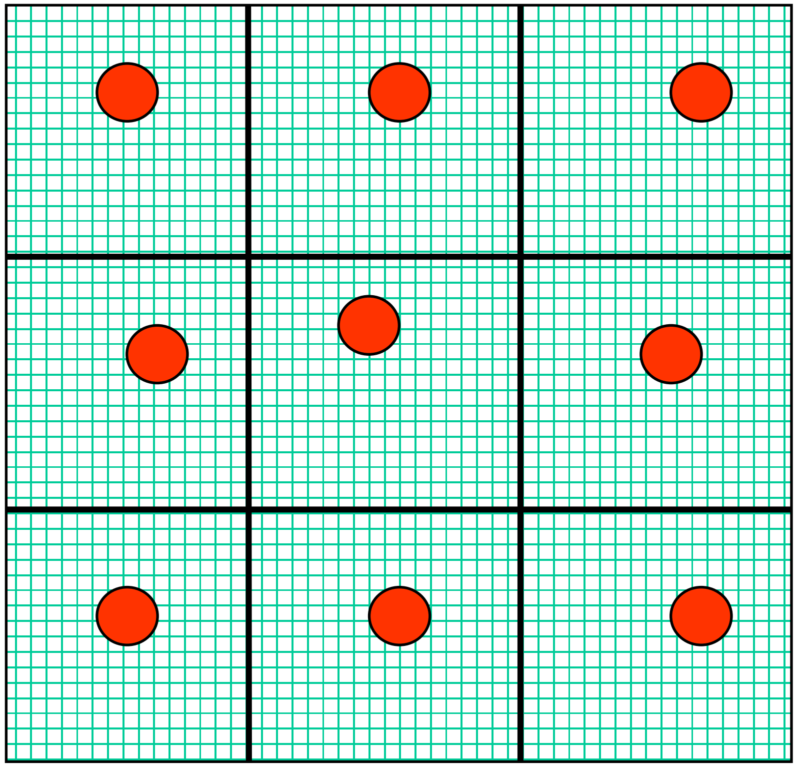
at *the right moment*

In the late sixties, Bell Labs create the first CCD camera, which allows the emergence of HS wave-front sensor (it is probably one of the first scientific use of a CCD !)



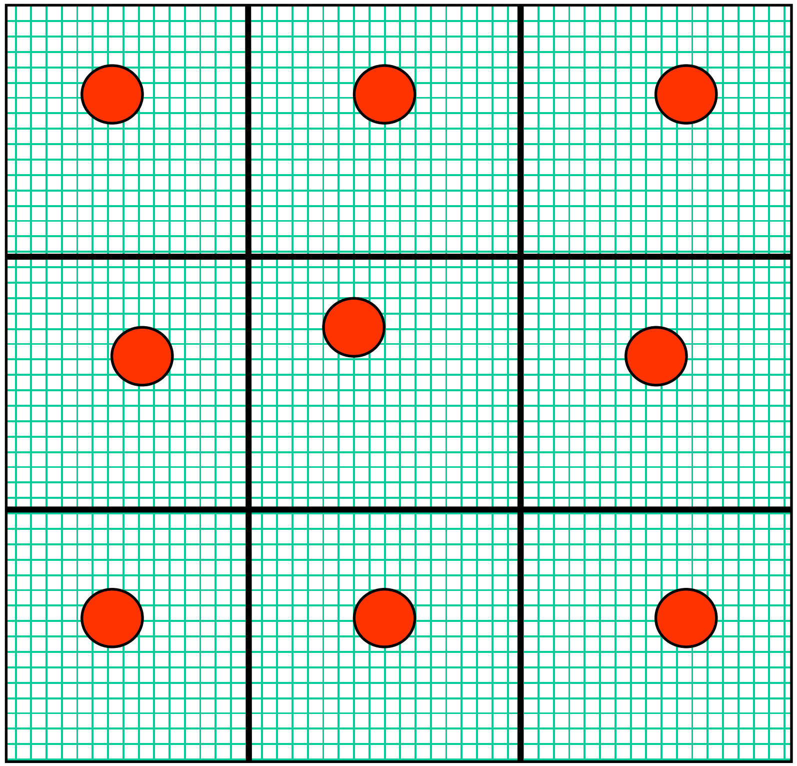
Hartmann-Shack: centroiding evaluation (1)

Definition
of areas of research



Measurement of x and y coordinates of each focal spot in each box leads to the evaluation of the local gradient in two directions

Hartmann-Shack: centroiding evaluation (2)



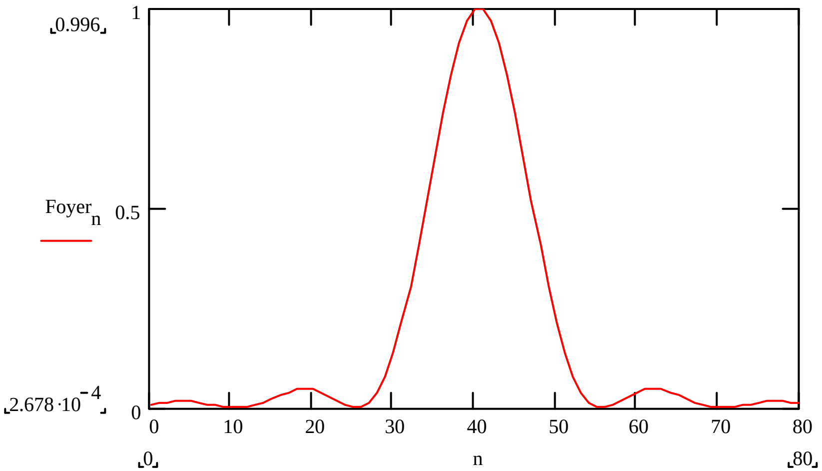
Limitation in dynamic: a focus point can’t approach a frontier

Hartmann-Shack :
a very (too?) simple geometrical model

Geometrical description implies

- -local independency of sub-pupils
- -a local wave-front only tilted at the scale of the sub-pupil

In fact, you have an Airy spot at the focus of each microlens, with a large extension which leads to cross-talk



J. Primot, Opt. Com., 2003

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Hartmann-Shack: summary

Created by Shack for astronomical purpose (low light level)
Basic idea : the use of a grid of microlenses

-the career frequency appears only in the common focal plane of μlenses
-all the energy is collected (sensitivity)

A specific property :

- *imaging ability in the common focal plane*

A lost property :

- *sensitivity is no more continuously adjustable*

A very didactic geometrical model :

-however, you have to be careful, as it is a rough approximation for metrological purpose

A set-up greedy in terms of pixels:

- *typically 8 by 8 pixels (or 16 by 16) by measurement point*

Manufacturers :

- *Imagine optic (F), Wavefront science (US), Thorlabs (S)*

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A second cousin : The Modified Hartmann Mask

Compared with HS, the MHM exhibits two specific properties:

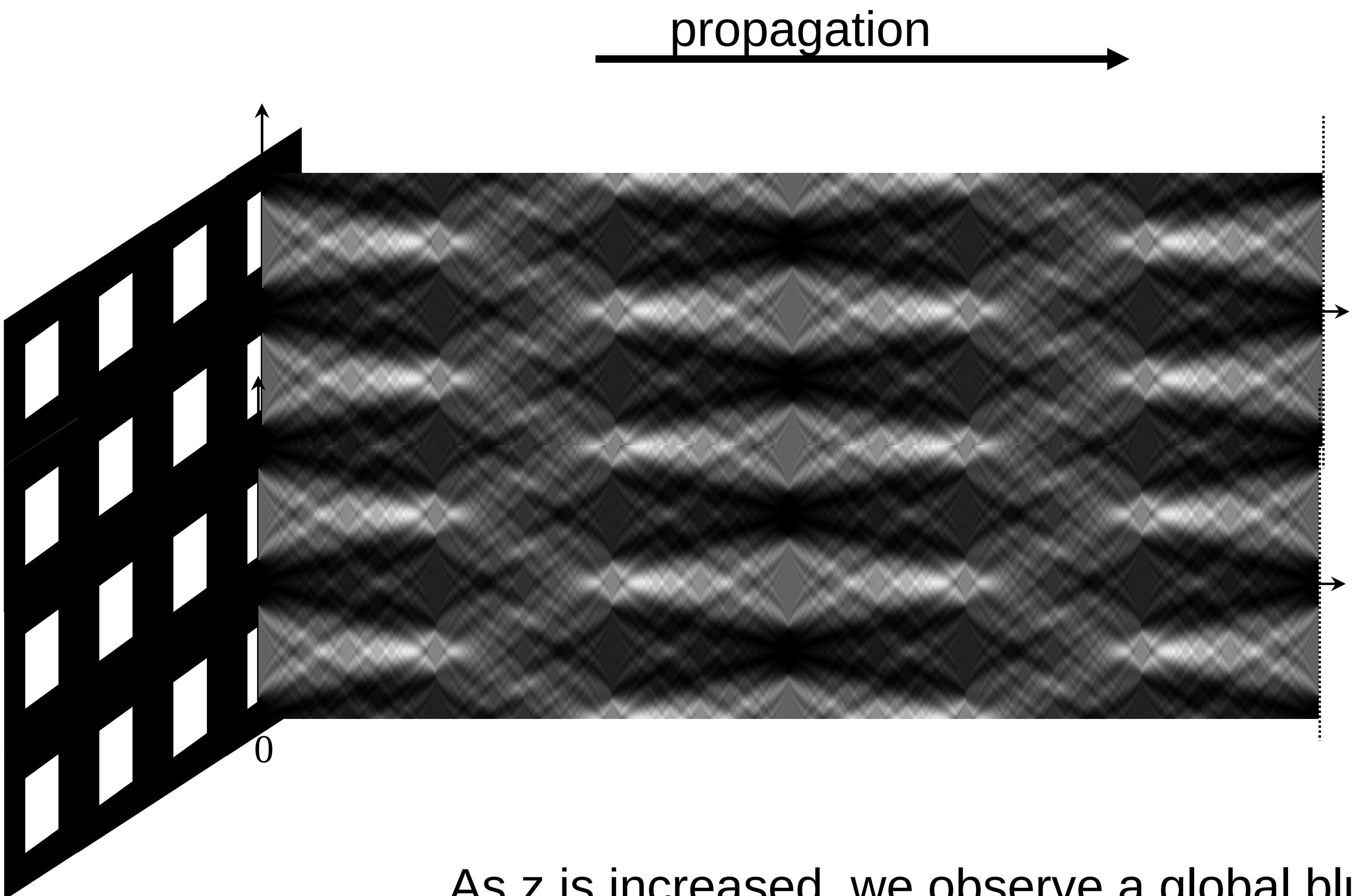
-first, it allows to recover the adjustable sensitivity

-second, it allows the optimization of the number of pixels needed for one measurement point

The set-up (the way to introduce the career frequency) is based on a new optical property:

the *non-diffracting arrays*

Hartmann Screen and diffraction (1)



As z is increased, we observe a global blur of the spots, due to global diffraction

Monochromatic illumination

Wavelength is going down

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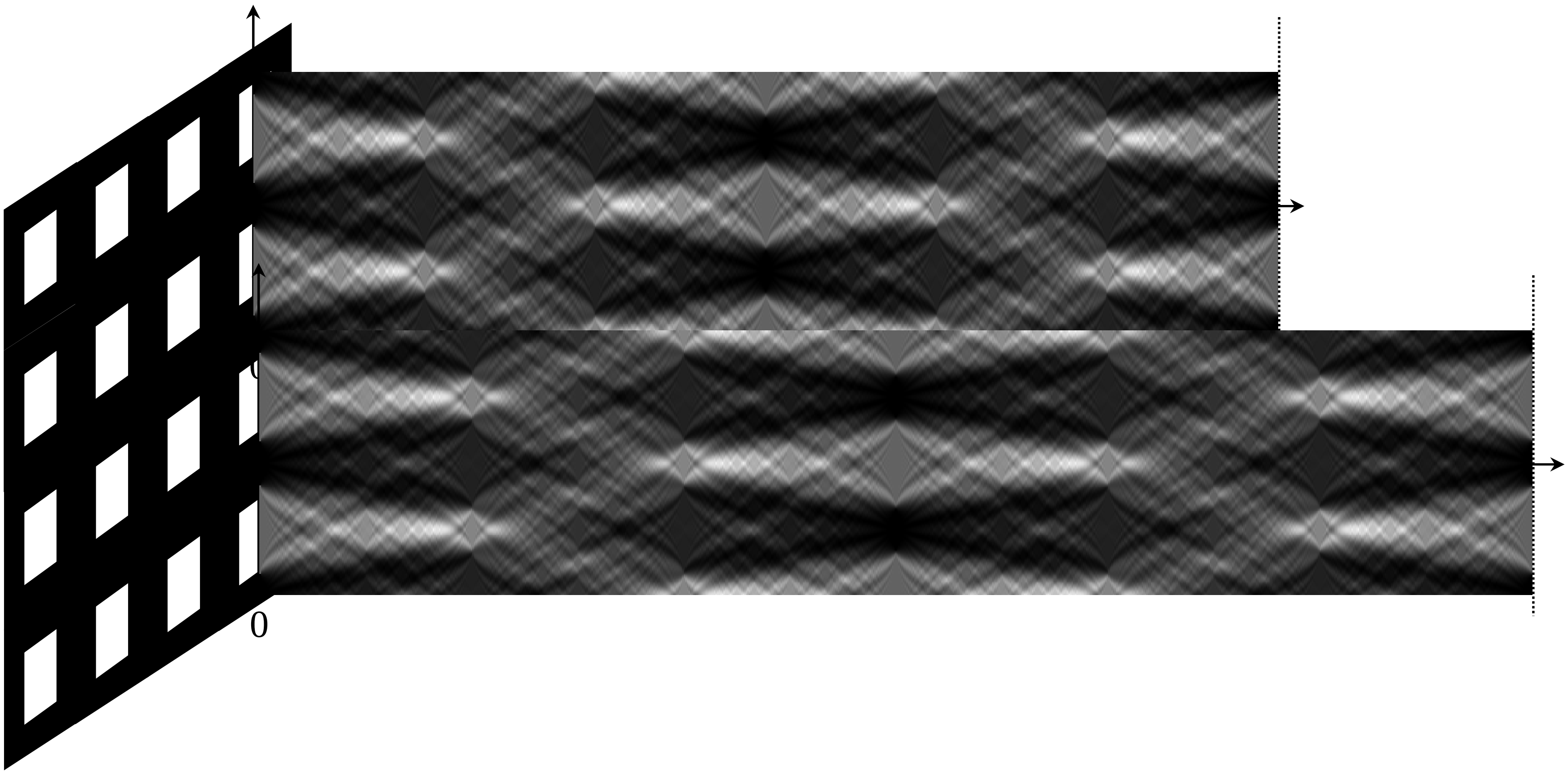
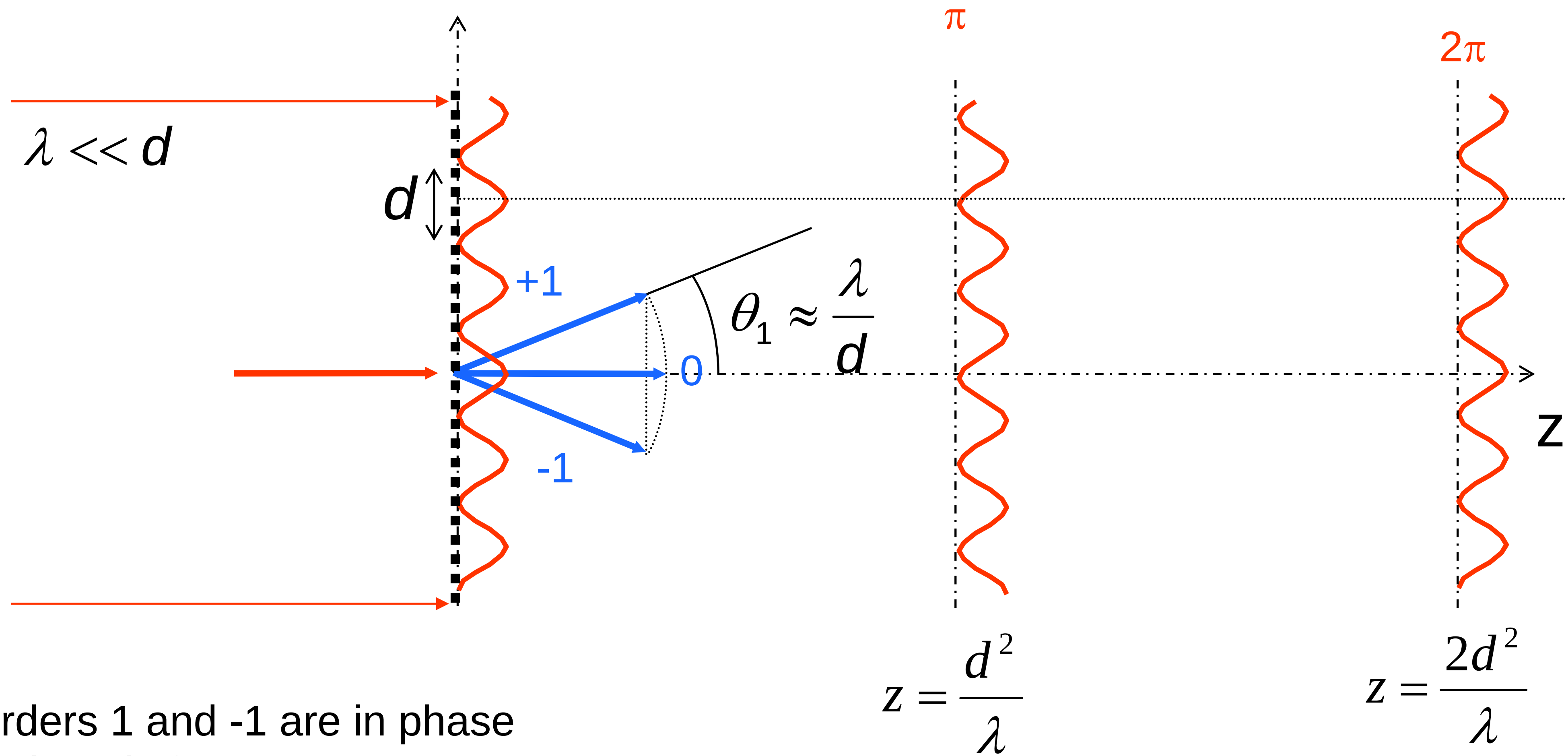
Hartmann Screen and diffraction (2)

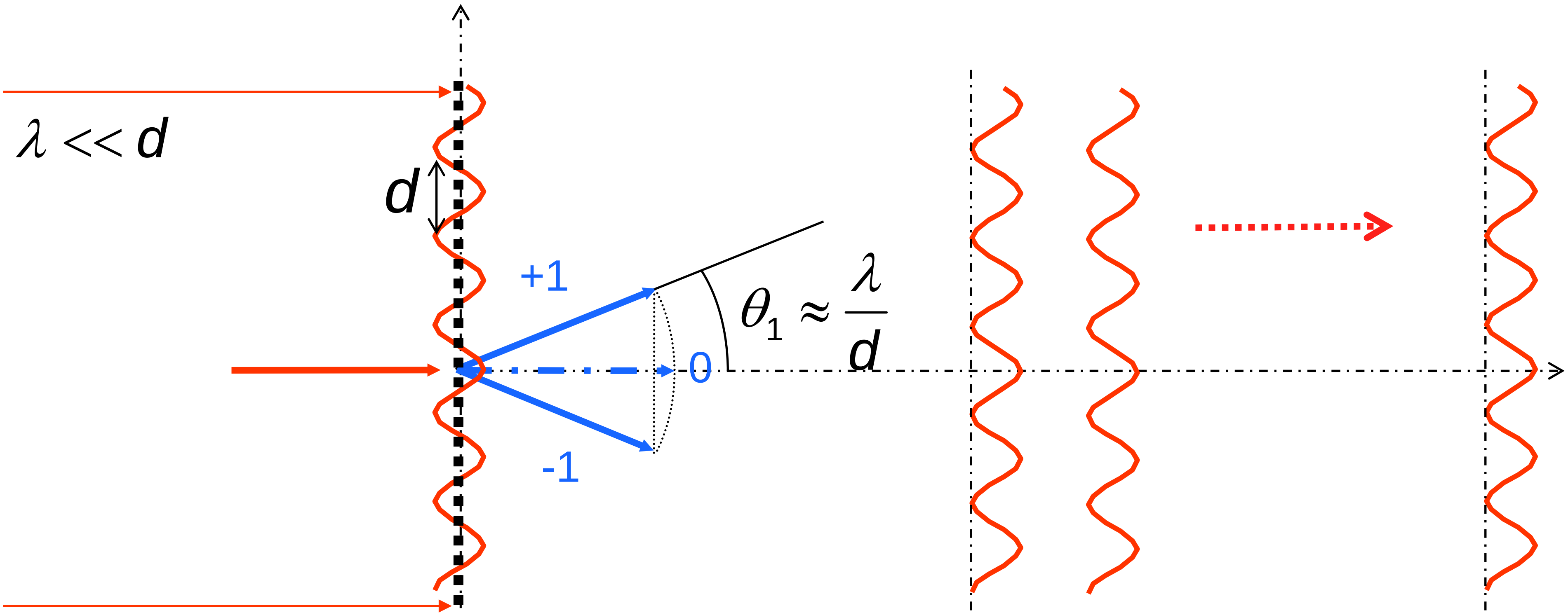


Talbot phenomenon (1836)
Explanation with a sinusoidal grating in irradiance



Orders 1 and -1 are in phase
Order 0 is faster
But periodically, the delay between (1,-1) and 0 is equal to lambda

Talbot phenomenon (1836)
Explanation with a sinusoidal grating in amplitude



Order 0 is cancelled
Orders 1 and -1 are always in phase
So, the irradiance profile is invariant by propagation

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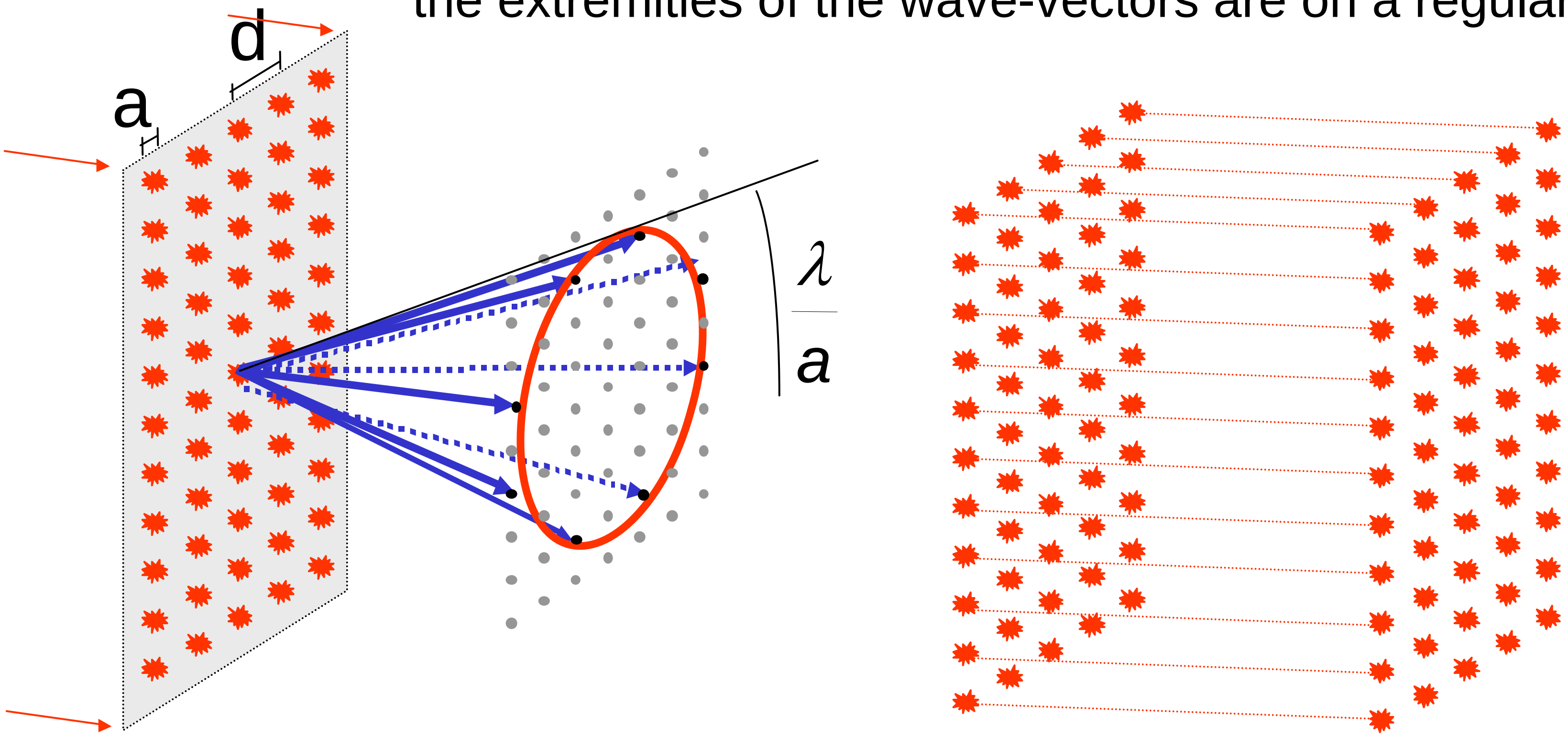
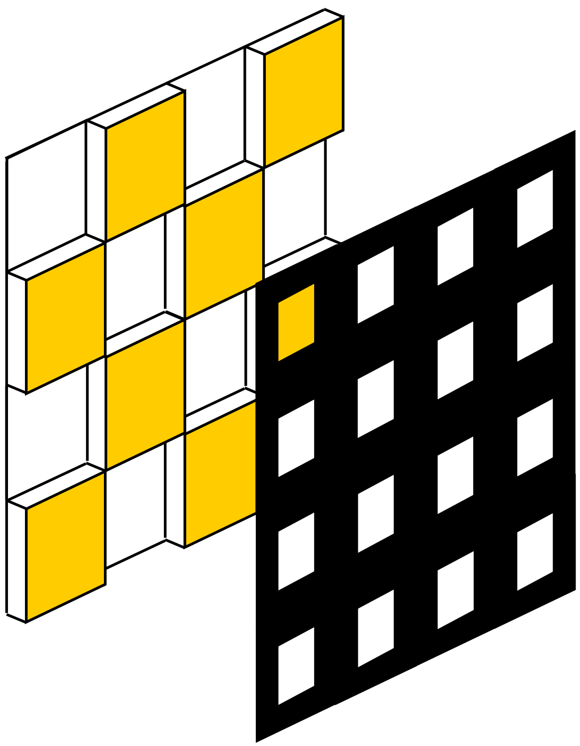
Non-diffracting array
Construction

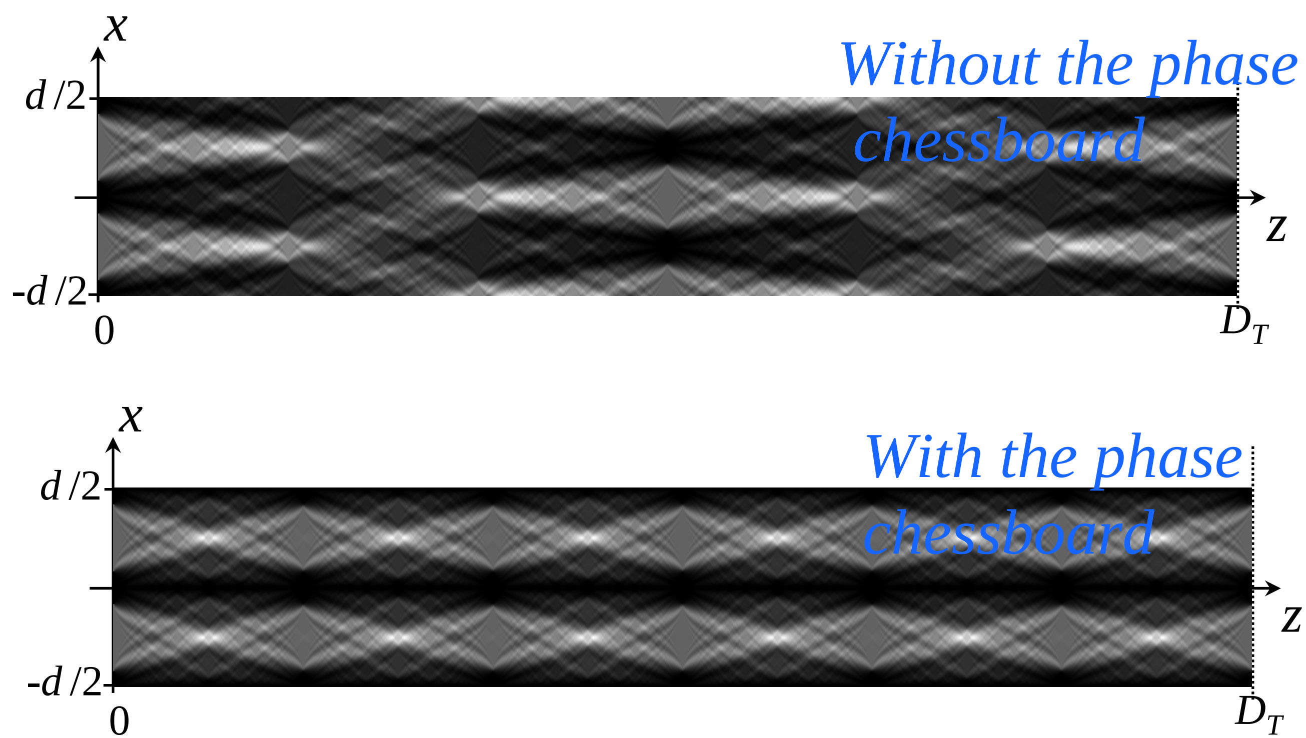
A bi-periodic grating diffracts orders,
the extremities of the wave-vectors are on a regular grid



An NDA is produced if you chose a grating with a profile such as it only diffracts orders, with wave-vectors at the intersection of the regular cartesian grid and a circle (a lot of solutions are available)

The simplest non-diffracting array: the Modified Hartmann Mask

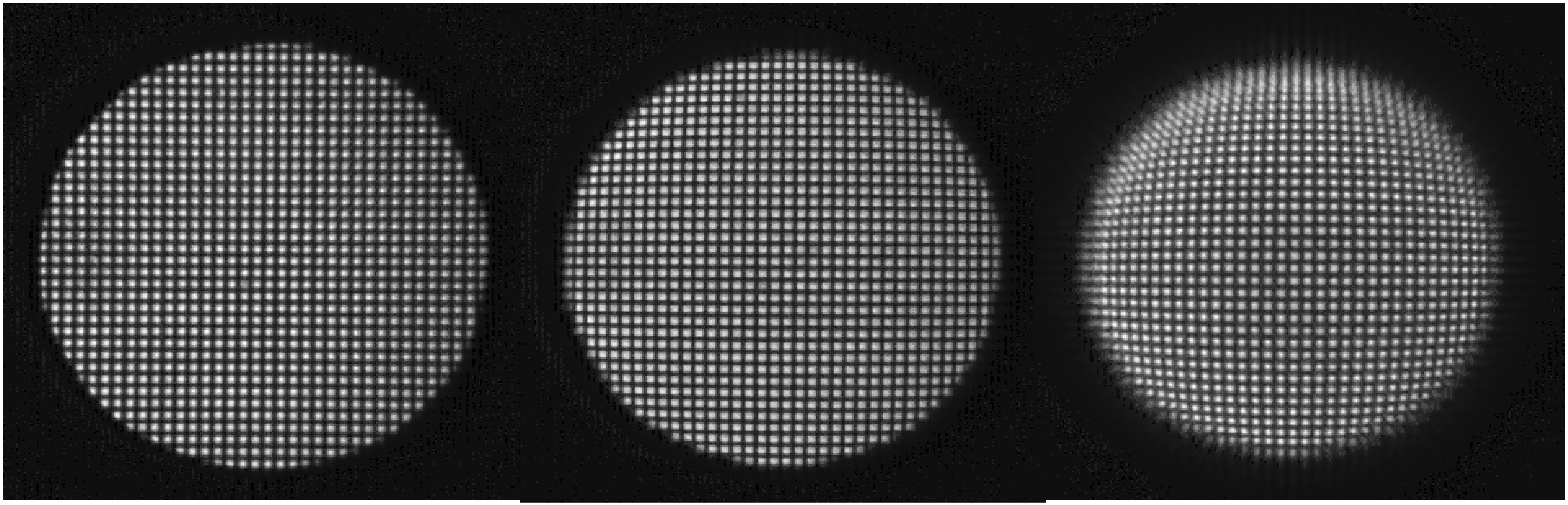




The Hartmann screen is completed by a phase chessboard with a double period and an amplitude of $\lambda/2$

Invariance by propagation, the basic tool of the engineer in optics

As the energy is pseudo-guided, you have a behavior consistent with ray-tracing => interferograms look like spot diagrams



Four-wave lateral shearing interferometry

Modified Hartmann Mask : summary

Deduced from Hartmann screen, for metrological applications

Basic idea : invariance by propagation

- introduce a career frequency insensitive to propagation

Specific properties :

- fringes are sinusoidal in the main directions (Shannon)

- sensitivity is continuously adjustable, beyond the limitations of Hartmann screen

A didactic geometrical model :

-similar to ray-tracing

A simple interferometric model :

- interference of 4 tilted replicas of the impinging wave-front

Less greedy than HS, more than Curvature :

- typically 4 by 4 pixels by measurement point (limit is 3 by 3 pixels)

Manufacturer :

Phasics (F) SID4 (sometimes called four wave lateral shearing interferometer)

Irradiance Transport Equation (ITE)

Curvature Sensor

« Variations on a Hartmann thema »

Geometric versus Interferometric

Wave-front reconstruction from derivatives,

Wave-front sensing : perspectives

Modified Hartmann Mask : the missing link ?

Two possible descriptions available

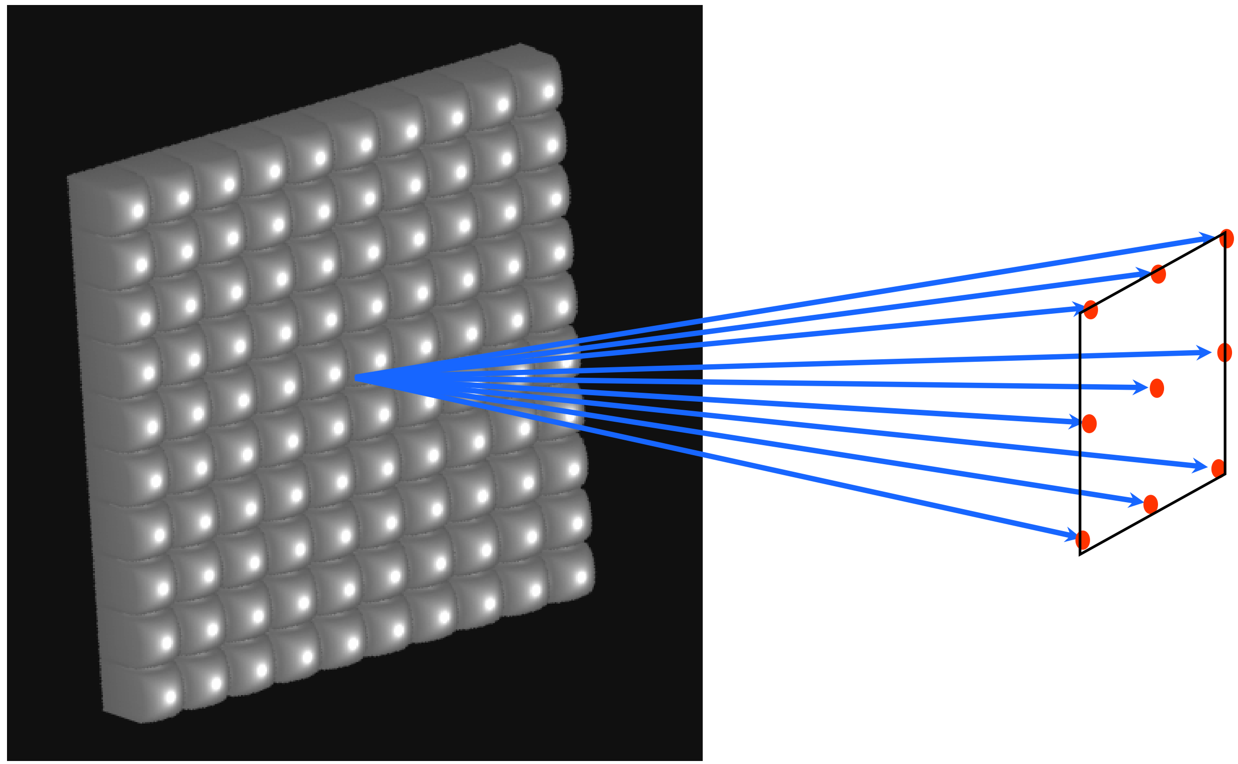
First : A geometric ray-tracing model,
rays are deviated with respect to the local tilt of the wave-front

Second : (necessary to explain the invariance by propagation)
A grating diffracts four tilted replicas of the impinging wavefront
The interference of these 4 replicas leads to a distorted interferogram

In fact, you have always the two descriptions available for each wave-front sensor

Geometry <u>and</u> Interferometry

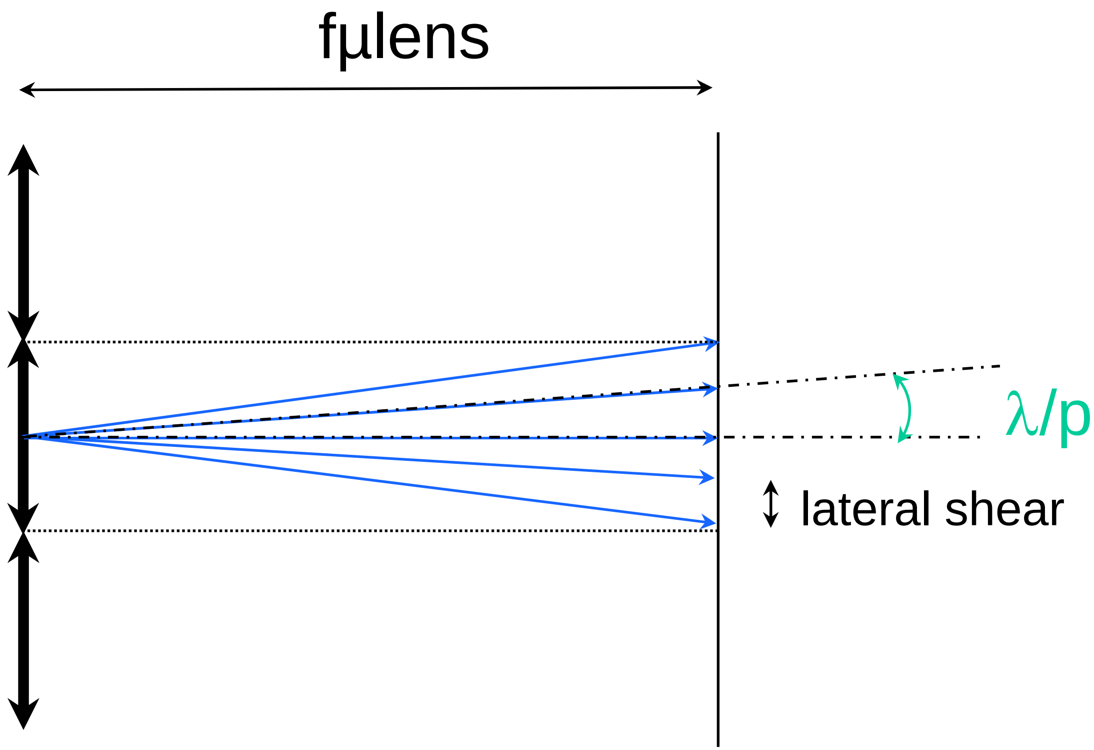
Hartmann-Shack is a grating interferometer (1)



µ-lens grid ➔ bi-periodical phase grating

F. Roddier, « variations on a Hartmann theme », Opt. Eng.

Hartmann-Shack is a grating interferometer (2)



lateral shear << sub-pupil size

Irradiance Transport Equation (ITE)

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How to process « Hartmanngrams » ?

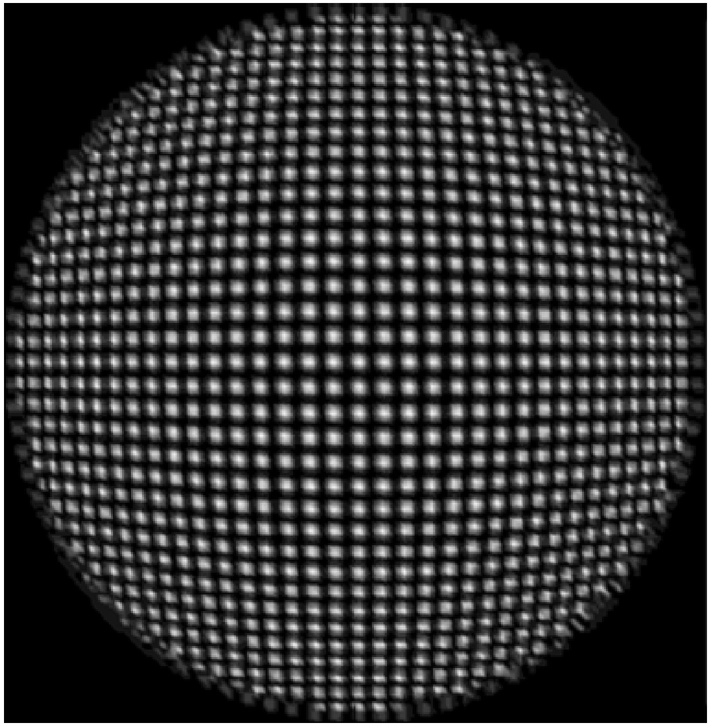
For all the members of the family, the basic principle is the same : the « Hartmanngram » (a bidirectional grid of points) is demodulated to obtain derivatives in different directions

In fact, signal processing is mainly independent of the set-up, even if you have to take into account specificities (compression and so on)

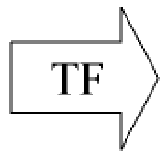
A first approach : the evaluation of the local coordinates of the spot, with a first step of definition of areas of research (cf. HS classical processing).

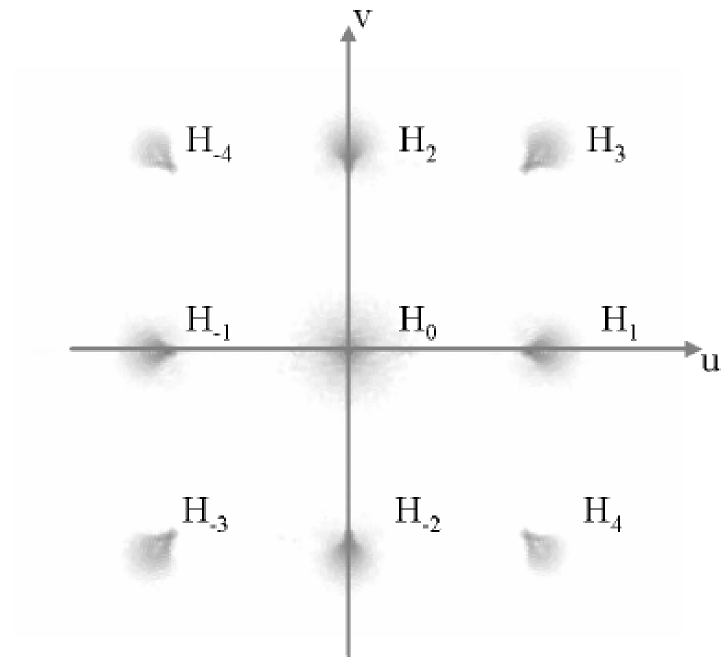
Proposition of a new approach : global demodulation by Fourier Transformation (analogy with FM/PM)

Let’s begin with MHM (1)



Interférogramme obtenu si W est
une aberration sphérique





Valeur absolue de son spectre

Let's begin with MHM (2)

In this example, the x-derivative of a spherical aberration is a coma

The harmonics is in fact the complex amplitude you would have observed at the focus if you were faced with the x-derivative of the analyzed phase

So, you have to :

select the harmonics in the x- (y)direction,

center it,

do an inverse Fourier transform,

extract the phase of the restituted complex amplitude (x-(y)derivative)

extract the amplitude of the restituted complex amplitude (scintillation)

General process :

1-Fourier Transformation of the hartmanngram

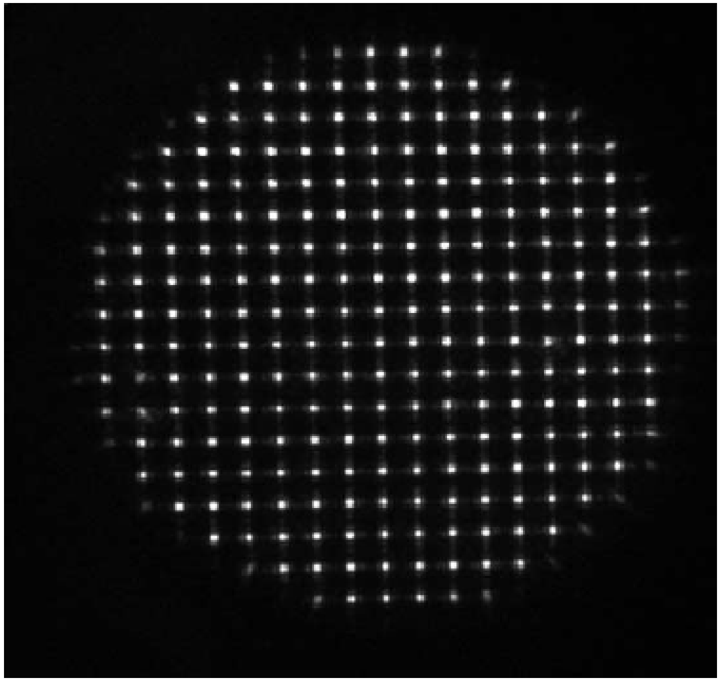
2- Selection of one harmonic, windowing, centering

3- FT^{-1}

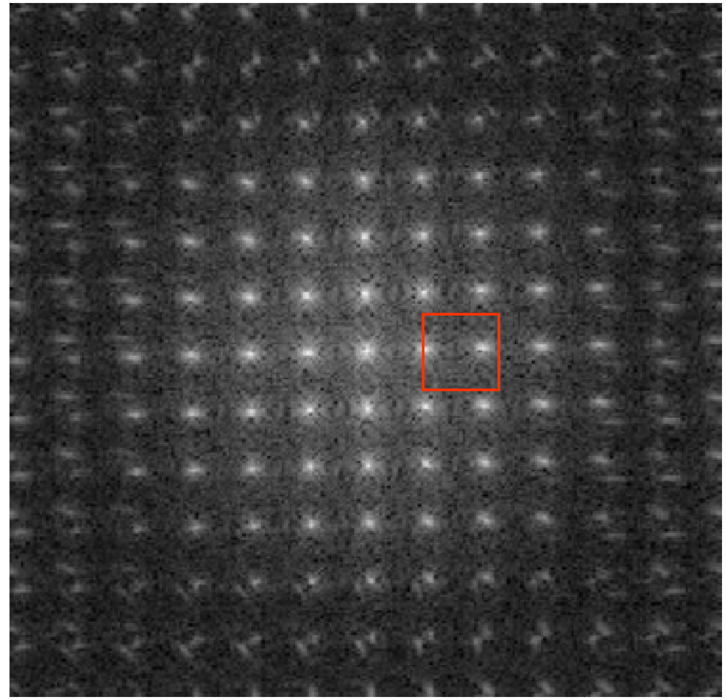
5- N derivatives are obtained by selection of N harmonics in different directions

for example, with MHM, you can obtain x_der, y_der, (x+y)_der, (x-y)_der

Hartmann-Shack FT analysis



Shack-Hartmanngram



FT of the fringes

Windowing of one harmonic
And so on…

Notice : the high compression ratio
(typically 16 by 16 pixels for a spot of nearly 2 by 2 pixels)
implies that the umber of points for the x-derivative is low
(the size of the window is small with respect to the size of the Hartmanngram

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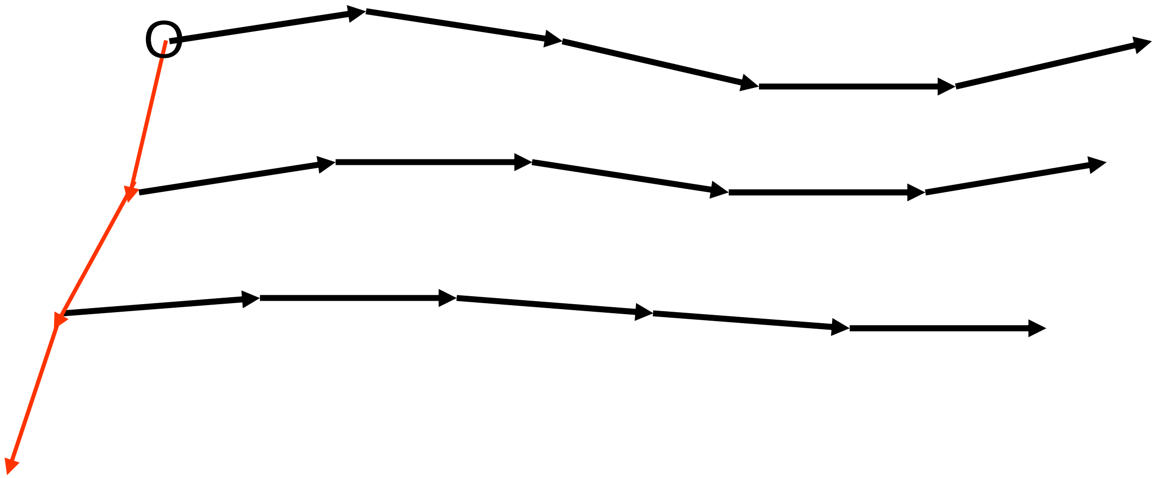
Phase reconstruction (1)

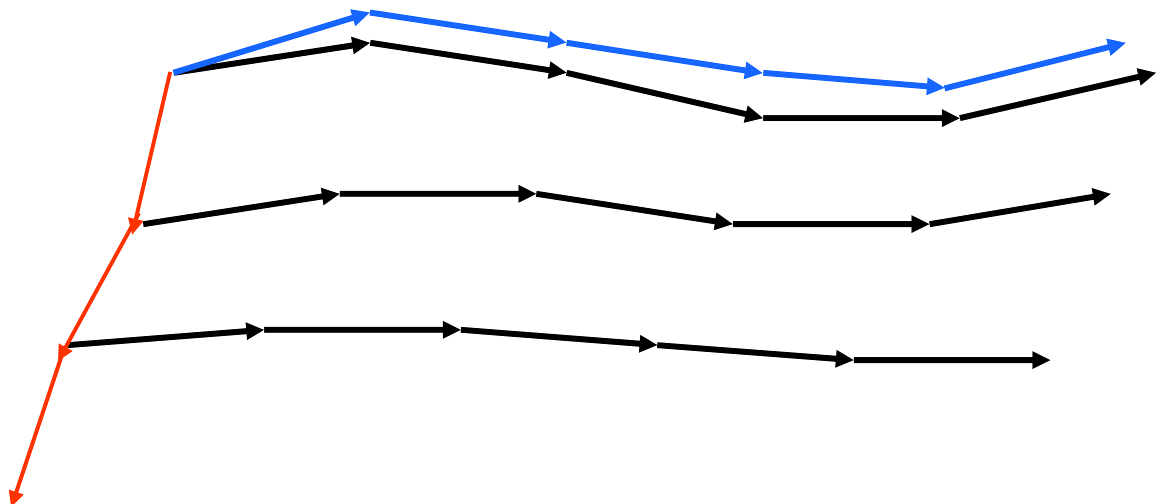
Why do you need two derivatives? (at least)
“Mathematically” speaking, you just need one derivative (for example x_der) for all the support, and one derivative (y) for only one column



Reconstruction of F(x,y)
Knowing dF/dx(x,y)

Phase reconstruction (2)

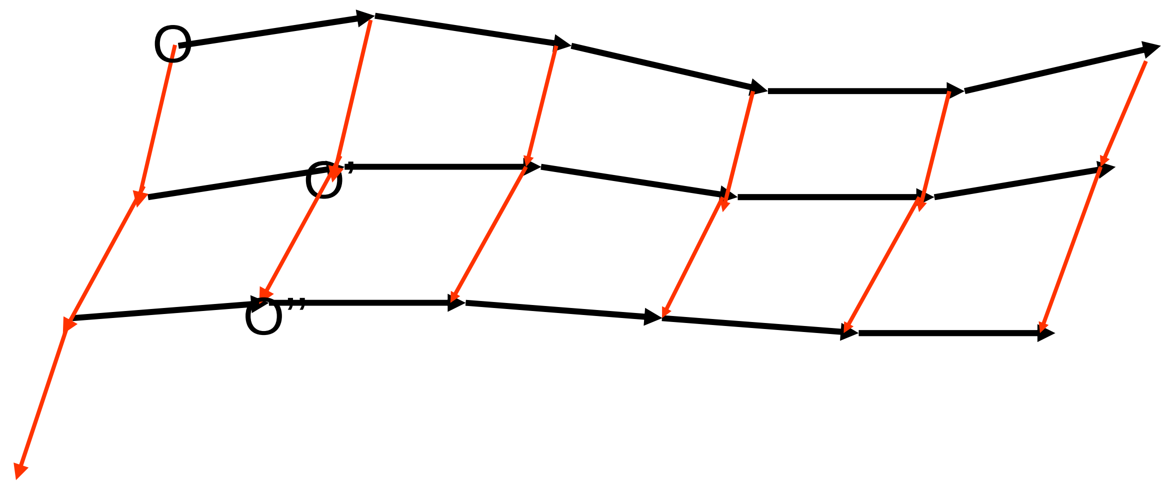
Reconstruction point by point is a processus in which measurement error for the x-derivative is propagating



After the first step, the most probable position is the arrival point, which is affected by the error,
So, this first error will be present in all the consecutive points

Phase reconstruction (3)

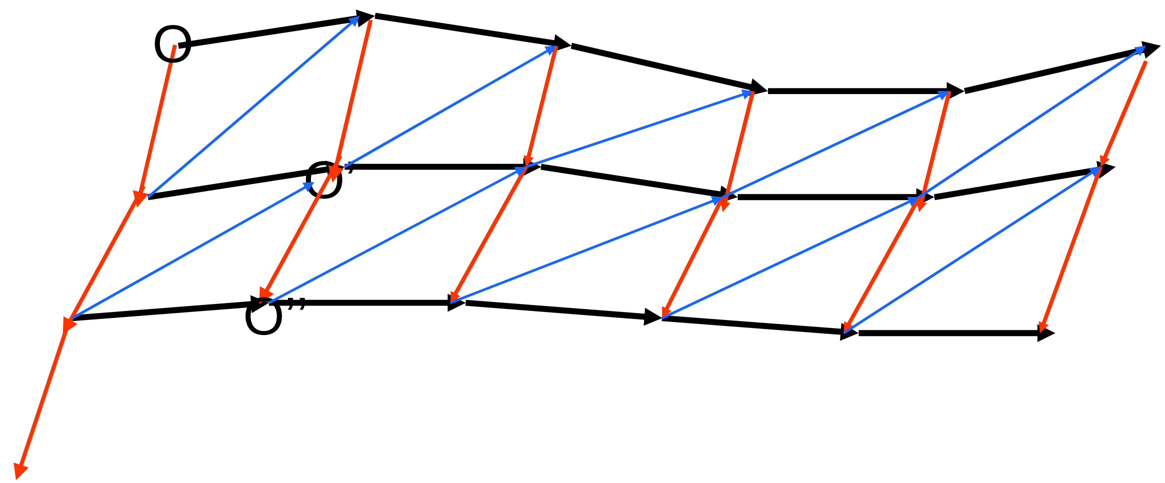
Using two derivatives (x and y) is a good way to stiffen the reconstruction
You have at least two ways to go from one point to another
For example, for two consecutive points in a diagonal, you can make a step in x-direction, and then in y-direction, or a step y and then step x



At least two ways implies a *least squares* approach to minimize the error

Phase reconstruction (4)

Using three derivatives (for example x, y, and x+y), with a least squares approach, is a better way to stiffen



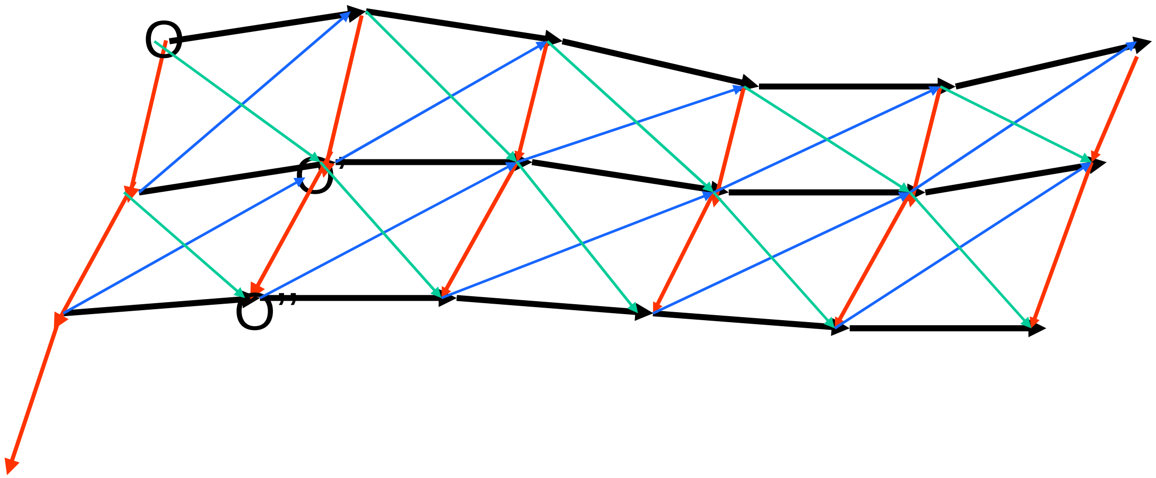
Phase reconstruction (5)

Let's try 4-derivatives (x, y, x+y, x-y)

the noise is reduced by a *(non-negligible)* factor of 1.2, compared with a two derivatives approach

However, the method is more complicated and slower

So, the number of derivatives to be considered depends on your application



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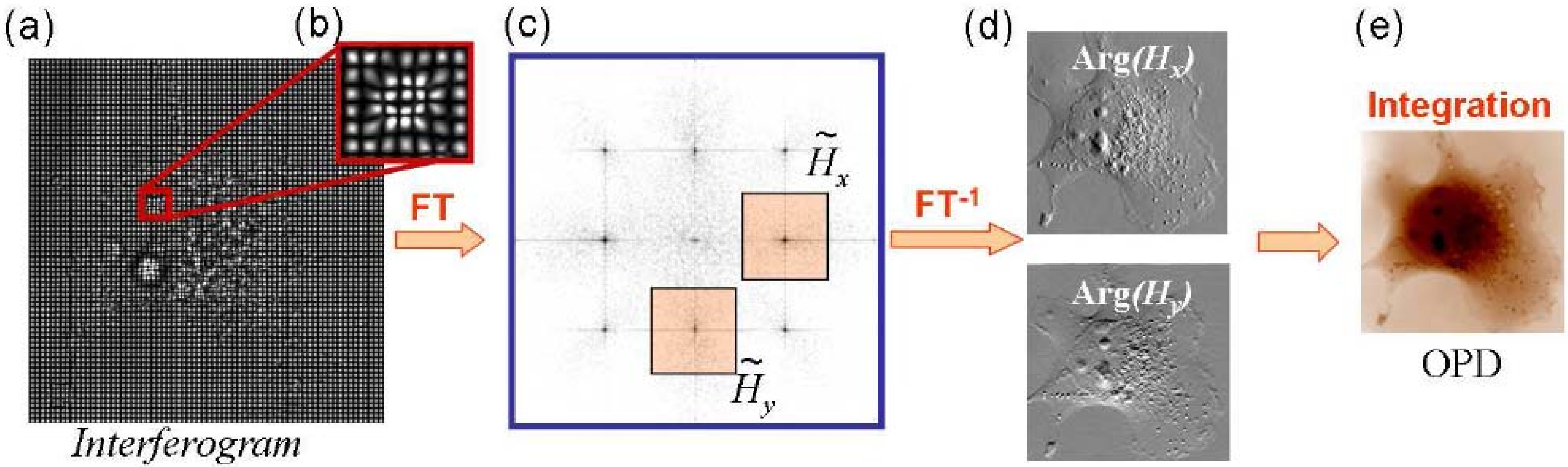
The future, phase imagery,
a link to Zernike phase contrast



P. Bon et al., Opt.Expr., July 2009

CONCLUSION

Shearing interferometry is a large framework,

in which most of the wavefront sensors industrially developped

(Hartmann, Shack-Hartmann, lateral shearing interferometers, multi-lateral shearing interferometers, deflectometry, …)

can be included

Thanks to the Irradiance Transport Equation

Allowing a theoretical common description and so, a fair evaluation, in the different contexts of application