Principles of Adaptive Optics and new challenges for Extremely Large Astronomical Telescopes

-- which modal basis for phase description ? --

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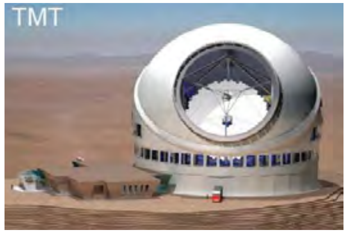
THE FRENCH AEROSPACE LAB

retour sur innovation

400 Years of Ground Based Astronomical Instrumentation

- From a few centimeter in 1609…

  …to 42 meters… in 10 years from now!



Galileo Galilei

Télescope W. Hershel D=1.24m

M. Wilson D=1.5m

M. Palomar D=6,05m

ESO D=3,6m

Keck D=10m

VLT D=8,2m

GEMINI D=8,1m

GTC D=10,4m

LBT D=8m

TMT D=30m

E-ELT D=42m

1609

1709

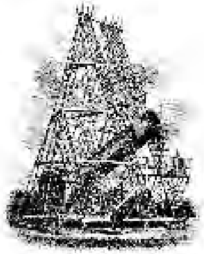
1809

1909

1953 AO Concept By Babcock

2009

Temps



Astronomical Imaging Through Turbulence

- Astronomy often implies high angular resolution imaging of distant objects: solar system bodies, stars, galaxies…

Photon story: Million years of peaceful travel but hectic last 100 µs…

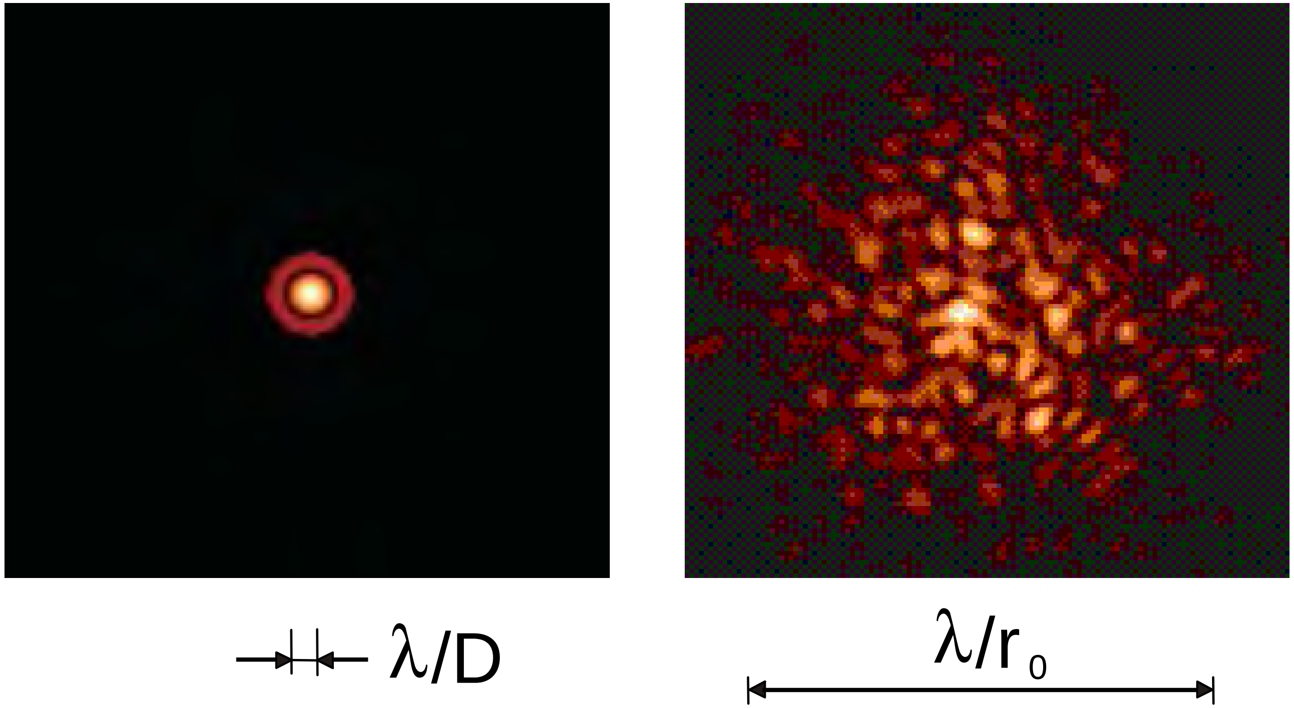
Resolution with Atmospheric Turbulence

> Isaac Newton in Opticks in 1730:
> “For the Air through which we look upon the Stars, is in perpetual Tremor … (…) The only Remedy is a most serene and quiet Air, such as may perhaps be found on the tops of the highest Mountains above the grosser Clouds.”

r_o : Fried param
λ/r_o : Seeing

No turbulence
Airy Pattern = diffraction limit

With turbulence :
PSF evolves at few ms time scale

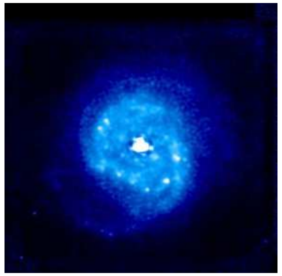


- **VLTs (10m telescope) have same resolution as a 60cm amateur telescope !!**

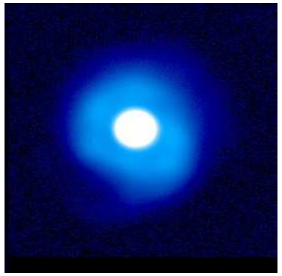
~ 1 arcsec instead of few 10 milli-arcsec diffraction limit !!

Adaptive Optics Principle

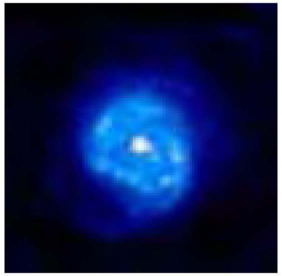
Object

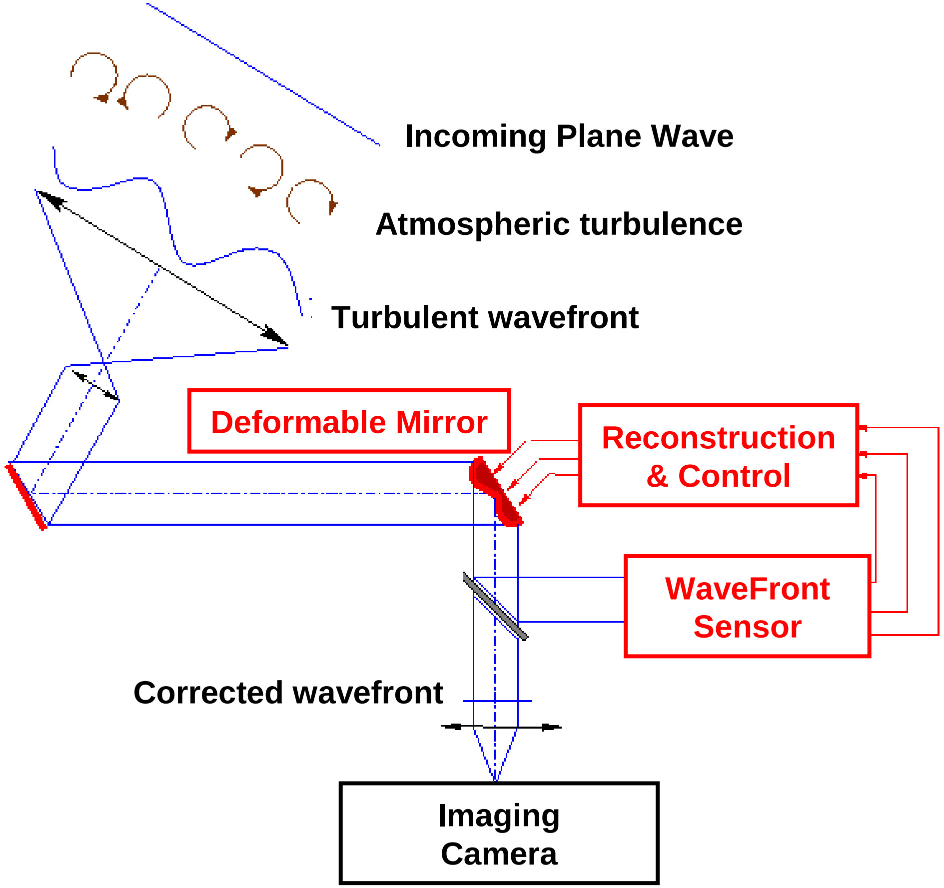


Uncorrected Image



Corrected Image





Incoming Plane Wave

Atmospheric turbulence

Turbulent wavefront

Deformable Mirror

Reconstruction & Control

WaveFront Sensor

Corrected wavefront

Imaging Camera

- Adaptive Optics allows to reach near diffraction limited angular resolution
- Main components: Turbulence ; Def. Mirror ; Wave-Front Sensor ; Rec. & Control

Outline

- ❑ Atmospheric Turbulence
- ❑ Deformable Mirror
- ❑ Wave-Front Sensing
- ❑ Reconstruction and Control
- ❑ From Very to Extremely Large Telescopes
- ❑ Illustrations :
  - ❑ Adaptive Optics Systems and Images
  - ❑ The European Extremely Large Telescope

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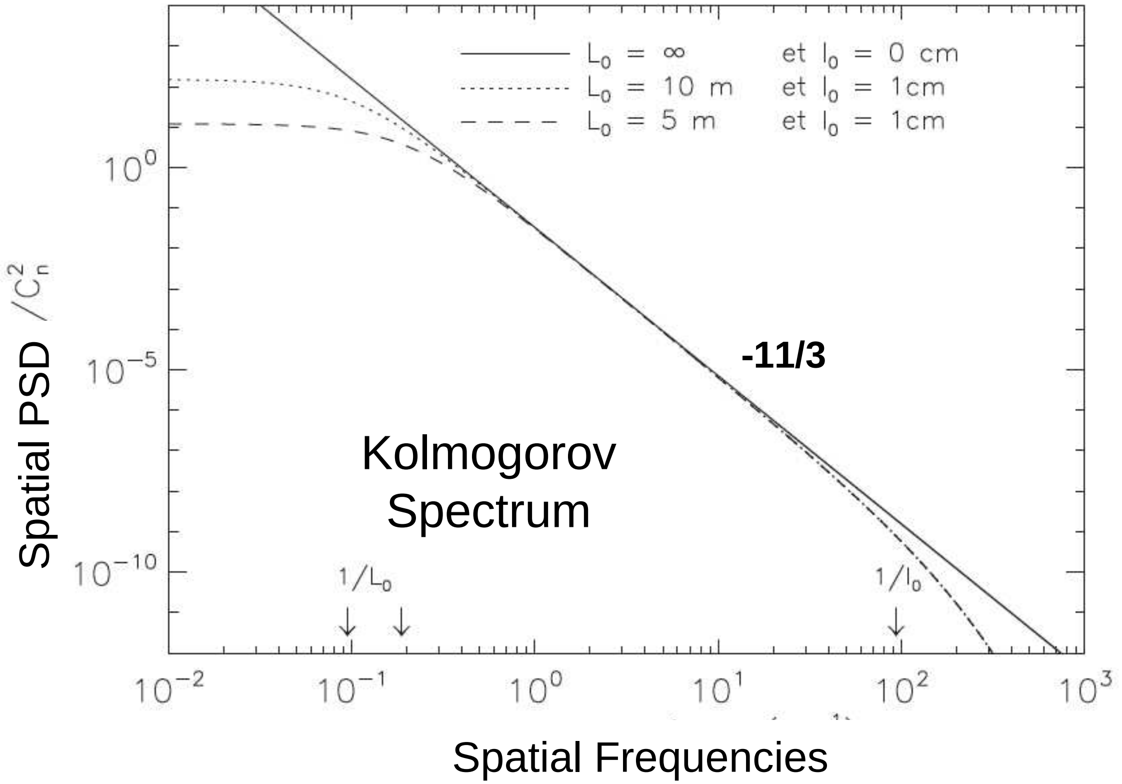
Turbulent Phase: Kolmogorov Statistics

$\Phi(\mathbf{r})$ = 2D map : Centered Gaussian Stationnary Process
Characterized by it Spatial Power Spectral Density (PSD)



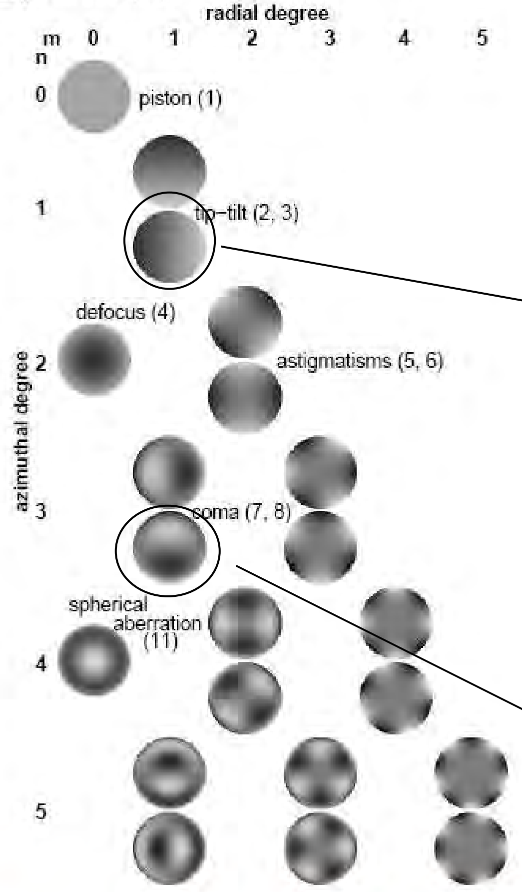
Leonardo da Vinci ; circa 1500

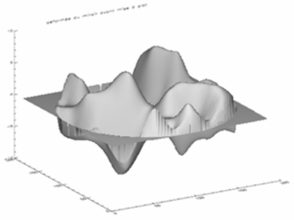
Statistics well known with simple power law for PSD
but manipulation of 2D continuous phase maps is difficult…
Solution: expansion of discrete set of modes, discretization in space (zonal)

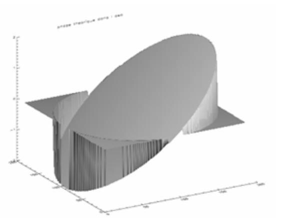
Turbulent Phase: Modal Expansions -- Zernikes

Decomposition on a modal basis $\{Z_i\}$:

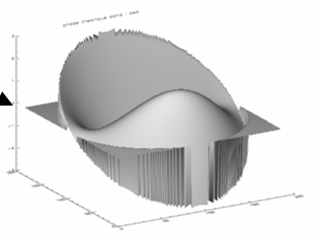
$$\phi(\mathrm{r}) = \sum_i \phi_i Z_i(\mathrm{r})$$







$Z_2(\mathbf{r})$



$Z_7(\mathbf{r})$

Radial Degree $n \nearrow \iff$ Spatial Frequency $\nearrow$

2D phase $\Phi(\mathbf{r})$ now characterized by vector of nz components: $\boldsymbol{\Phi} = \{\Phi_i\}$

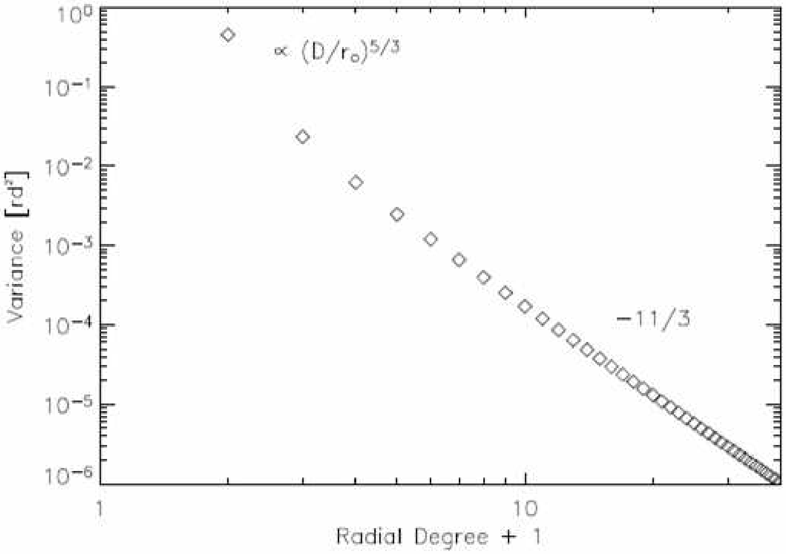
Open questions:

- Turbulence statistics on Zernike coefficients?
- Number of modes nz for a good characterization?

Turbulent Phase: Modal Expansions – Zernike Polyn.

- Kolmogorov spectrum $\longrightarrow$ covariance matrix C_ϕ.
- $C_\phi(j,j) = \sigma^2_{\phi_j}$, energy distribution on Zernikes :

Given by analytical expression (almost diagonal but not quite)



Phase variance above radial order *n*$_o$:

$$\sigma^2_\phi(n > n_o) \approx 0.46\,(n_o+1)^{-5/3}\,(D/r_o)^{5/3}$$

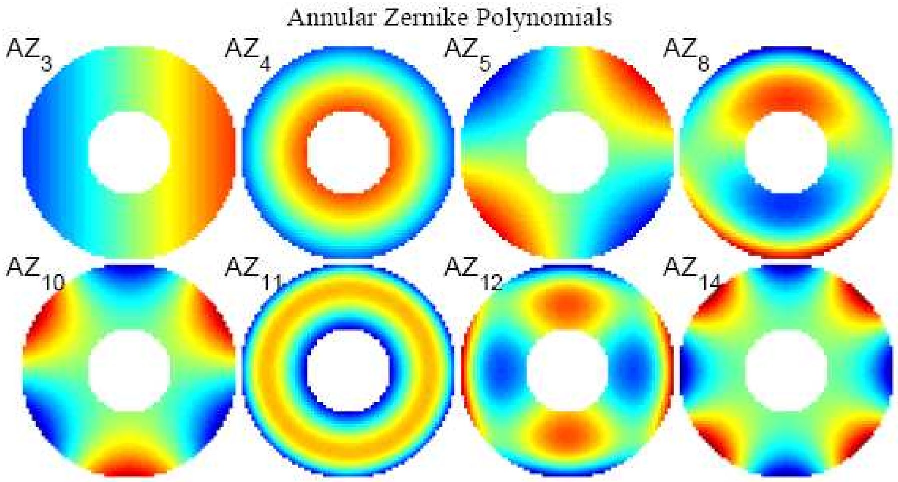
Choice of maximum radial degree ***n*$_o$** for turbulent phase representation

$$\sigma^2_\phi(n > n_o) \quad << 1 \text{ rd}^2$$

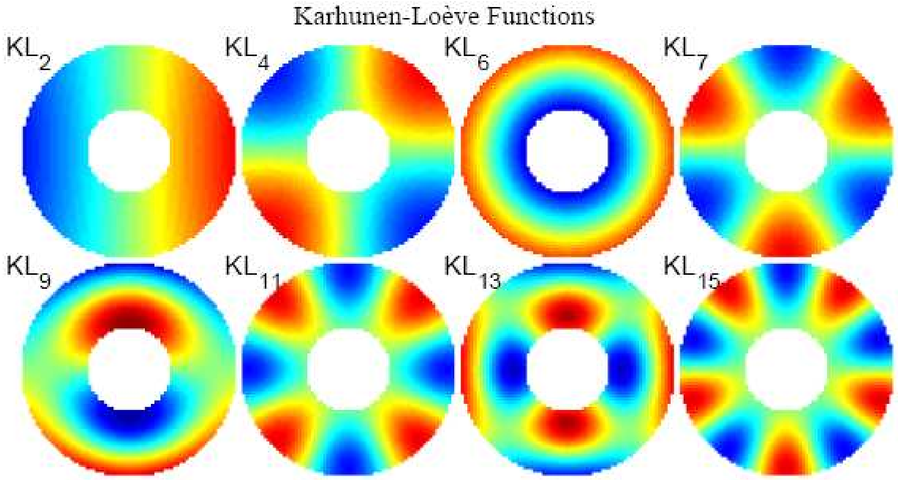
***n*$_o$** ~ D/r$_o$ a few 10 for VLT that is nz ~ $(D/r_o)^2$ a few 100 Zernike polynomials

Turbulent Phase: Modal Expansions – Karhunen Loève

$KL_i(\mathbf{r})$ are the statistically independent modes of turbulence: C_ϕ diagonal



Zernike properties:
- analytical expression of modes and covariance
- concentrates energy in low orders
- numerical calculation for very high order modes is tricky



KL advantages:
- similar in shape to Zernike pol.
- diagonal covariance
- concentrates even more energy
- numerical methods to compute modes and covariance,
- good numerical stability even for high orders

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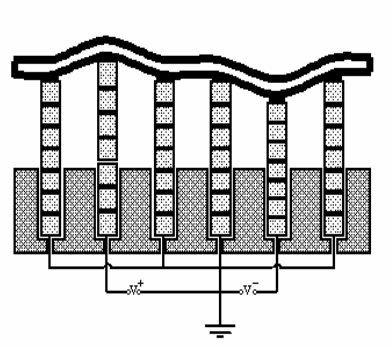
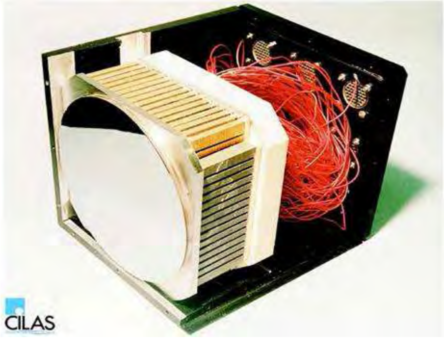
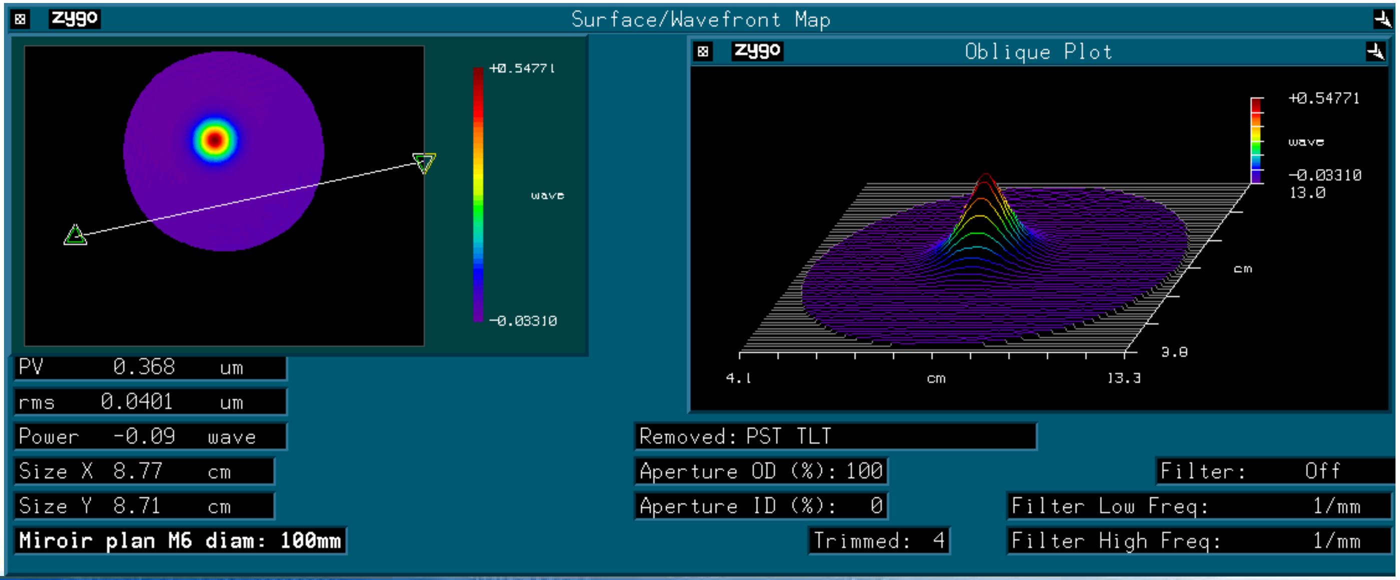
Deformable Mirrors (DMs)

- ❑ Thin optical plate
- ❑ Piezo-electric actuators driven by Nact voltages **u**



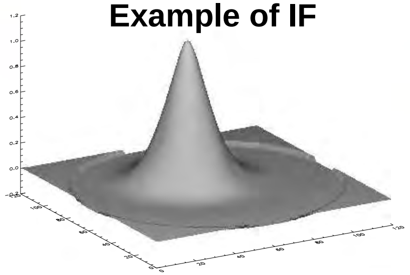


- ❑ DM characterized by its response to each actuator = Influence Functions (IFs) can be measured with interferometer (Zygo…)



PV 0.368 um
rms 0.0401 um
Power -0.09 wave
Size X 8.77 cm
Size Y 8.71 cm
Miroir plan M6 diam: 100mm
Removed: PST TLT
Aperture OD (%): 100
Aperture ID (%): 0
Trimmed: 4
Filter: Off
Filter Low Freq: 1/mm
Filter High Freq: 1/mm

Deformable Mirrors (DMs) : Correction Space


Example of IF

Correction phase $\boldsymbol{\Phi}_{\mathbf{corr}}$ spans a space of Nact dimension:

- Influence Functions $\{IF_i\}$ are a basis of the DM space

  Voltages $\{u_i\}$ = coordinates in IF basis :

  $$\boldsymbol{\Phi}_{\mathbf{corr}} \equiv \mathbf{u} = \{u_i\}$$

- Transposition in any other basis (Zernike, zonal…) given by:

  $$\boldsymbol{\Phi}_{\mathbf{corr}} = \mathbf{N}\,\mathbf{u}$$

  N influence matrix (columns of N give IFs in selected basis)

- Projection onto DM space:

  Best fit (Least Square sense) of a given phase **Φ** by the DM obtained with:

  $$\mathbf{u} = (\mathbf{N}^T\,\mathbf{N})^{-1}\,\mathbf{N}^T\,\boldsymbol{\Phi}$$

  Orthogonal projection of **Φ** onto the DM space

  in practice **Φ** has to be estimated from WFS data, in a rich enough space…

IFs are important for fine modeling of DM: specification, performance estimation…
However for preliminary AO modeling, DM IFs are often assumed to be
the first ~Nact Zernike (or KL) polynomials

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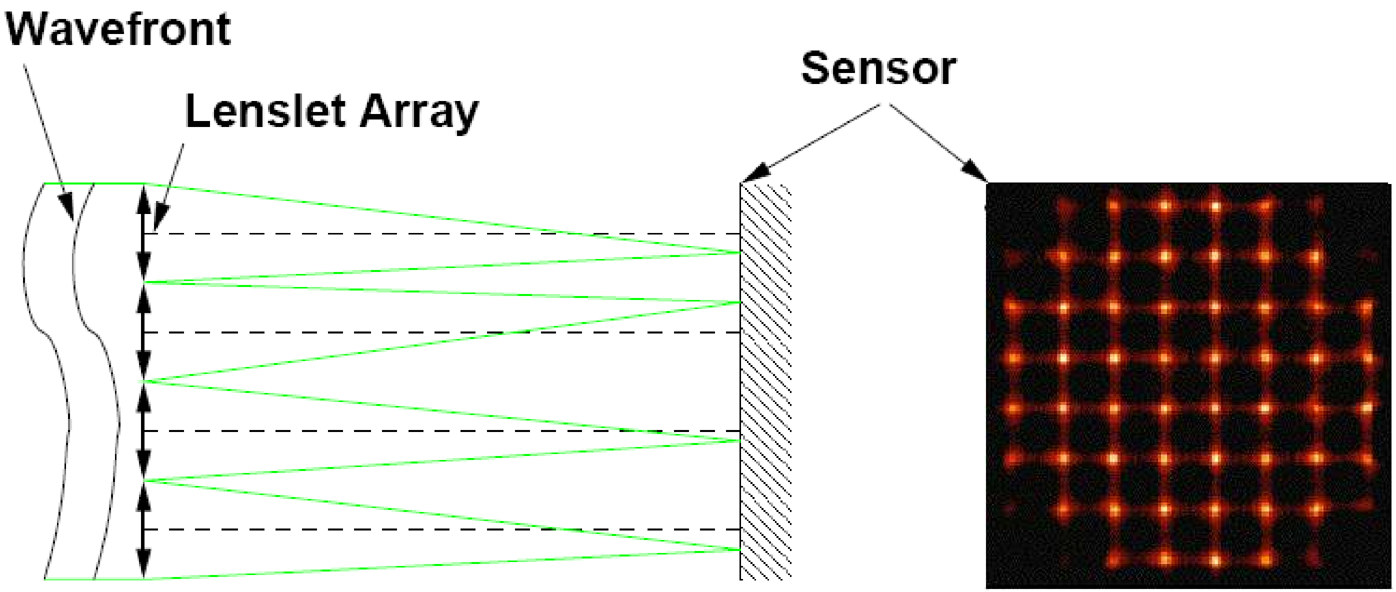
Shack-Hartmann Wave-Front Sensor (WFS)



K **sub-apertures** $\longrightarrow 2\,K$ **centers of gravity** $\{s_{x/y,i}\} = \mathrm{s}$

Principle: measurement of the wave-front local slope:

$$s_{x,i} = \frac{\lambda\, f}{2\,\pi\, S_i} \int_{S_i} \frac{\partial \phi(x,y)}{\partial x} \mathrm{d}x\, \mathrm{d}y + noise$$

WFS data are called “slopes”

- **Linear model :**

$$\mathrm{s} = D\phi + \mathrm{w}$$

WFS Linear Model

☐ **Linear model :**

$$s = D\phi + w$$

☐ **with for instance :** $\phi = \{\phi_j\}$ **and** $\phi(x,y) = \sum_{j=2}^{j_{max}} \phi_j Z_j(x,y)$

$$D_{j_{max}} = 2K \updownarrow \begin{pmatrix} \longleftarrow j_{max} \longrightarrow \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

☐ **If the phase is assumed in the DM space :** $\Phi = \{u_i\}$

$$s = DNu + w = D_{inter}u + w$$

$$D_{inter} = 2K \updownarrow \begin{pmatrix} \longleftarrow nact \longrightarrow \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

D_{inter} **is the so-called interaction matrix**

Linear operator / Matrix D : from Phase Space to Slope Space

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Wave-Front Reconstruction from WFS data
Least Square Solution

- Deterministic Least-Square [LS] fit of the measurement:

find $\hat{\phi}_{LS}$ minimizing $\|\mathrm{s} - D\hat{\phi}\|^2$

- Analytical solution :

$$\hat{\phi}_{LS} = \left(D^t D\right)^{-1} D^t\, \mathrm{s} = R_{LS}\, \mathrm{s}$$

- Limitations:
  - ☞ $D^t D$ generally ill conditioned (badly seen modes)
  - ☞ truncation of number of turbulent modes or TSVD required to limit noise amplification
  - ☞ no explicit account of Kolmogorov/noise statistics

not optimal: minimization of criterion on measurements

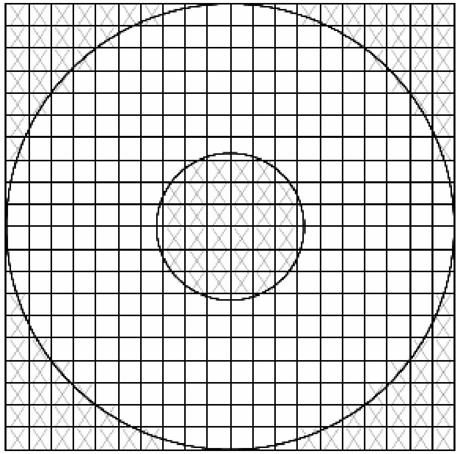
while the relevant quantity in imaging is the phase

- **Possible truncation strategies:**
  - **Reconstruction on limited number of Zernike pol.:** $D \equiv D_{j_{max}}$
  - **Reconstruction on DM space (that is on IFs):** $D \equiv D_{inter}$

    reconstruction matrix called in this case command matrix R_{com}

WFS Reconstruction: a tutorial case: description

- ❑ 20×20 sub-apertures: $K = 276$ valid sub-apertures ; $2K$ slopes



- ❑ $\mathrm{s} = D_{j_{max}} \phi + \mathrm{w}$ with $\phi = \{\phi_j\}$ $j \in \{2, j_{max}\}$ Zernike coefficients

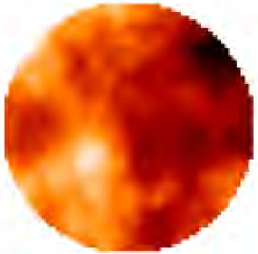
$$D_{j_{max}} = 2K \updownarrow \begin{pmatrix} \longleftarrow j_{max} \longrightarrow \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

nz = j_{max} - 1

WFS Reconstruction: a tutorial case: Least Square Rec.

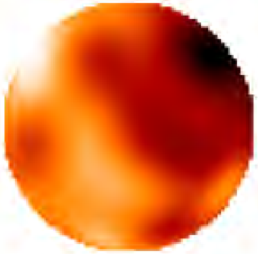
$SNR_{slopes} = 1.$, $K = 276$ sub-apertures

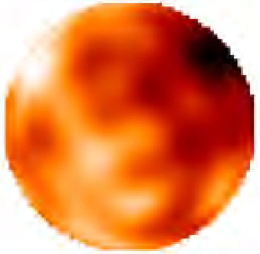
$$\hat{\phi}_{LS} = \left(D^t_{j_{max}} D_{j_{max}} \right)^{-1} D^t_{j_{max}} \, s$$


True Phase


TLS $j_{max} = 3$


TLS $j_{max} = 21$


TLS $j_{max} = 55$


TLS $j_{max} = 105$


TLS $j_{max} = 210$

Reconstruction quality highly dependent of j_{max}

Difficult to control the noise amplification on badly seen modes

WFS Reconstruction: a tutorial case: Optimal Rec.

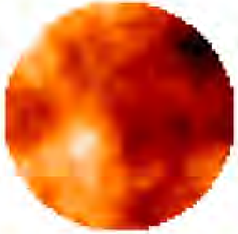
Optimal Reconstruction (Minimum Mean Square Error Estimator alias MMSE):

- ❑ **Uses a priori information on turbulence and noise statistics**
- ❑ **Allows reconstruction on infinite (very large in practice) number of modes**

$SNR_{slopes} = 1.$, $K = 276$ sub-apertures

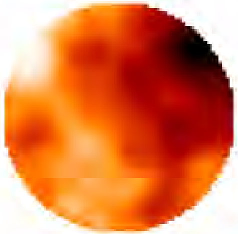
$$\hat{\phi}_{\text{mmse}} = \left(D_\infty^t C_w^{-1} D_\infty + C_\phi^{-1}\right)^{-1} D_\infty^t C_w^{-1} s$$

MMSE Reconstructor



True Phase

$[\sigma_\phi^2 = 3.0\, rd^2]$



MMSE

$[\sigma_{error}^2 = 0.7\, rd^2]$

TLS $j_{max} = 55$

$[\sigma_{error}^2 = 0.8\, rd^2]$

Optimal reconstruction: minimal phase error variance

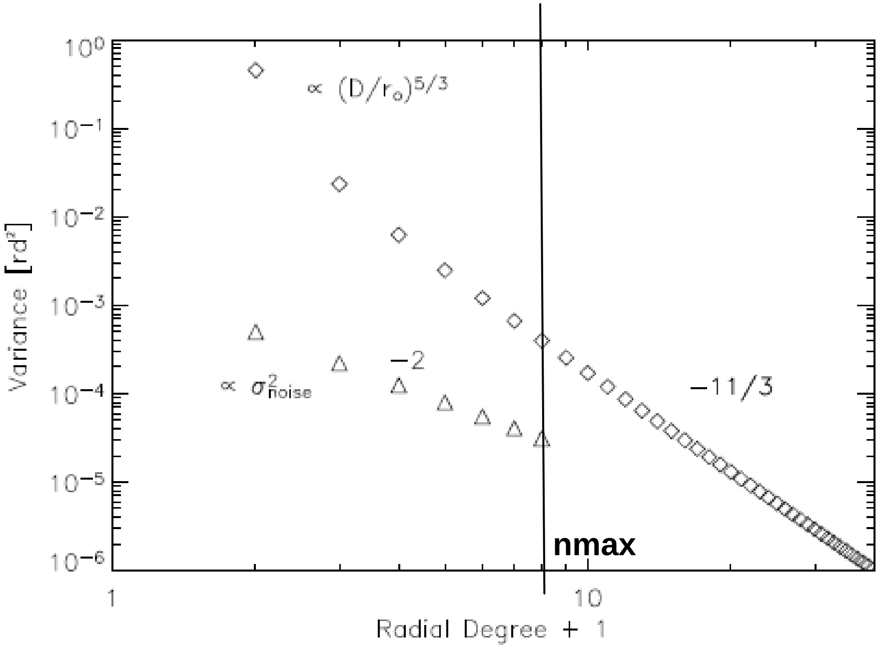
Best interpolation of the WFS measurents based on priors

Reconstruction on infinite number of modes thanks to regularization

Reconstruction on many modes limits the impact of somewhat arbitrary choice of the basis

WFS Noise Propagation onto Zernike coefficients

- ☐ $C_{\phi\ noise} = R\, C_{s\ noise} R^T$
- ☐ $C_{\phi\ noise}(j,j) = \sigma^2_{\phi j\ noise}$, noise propagated onto the Zernikes :



white noise on slopes + integration $\longrightarrow \propto (n+1)^{-2}$

With Least Square Rec. noise energy may become larger than turbulence if nmax/jmax too large
Optimal reconstruction overcomes this issue!

From Reconstruction to Control

Difficulty: turbulence, DM correction and WFS data are in distinct spaces
& WFS data are noisy

Two control strategies:

- **Standard** Least Square Rec. on restricted subspace to limit noise amplification: for instance on DM space to directly obtain control voltages

Advantage: simplicity, only calibration needed = interaction matrix

But not optimal…

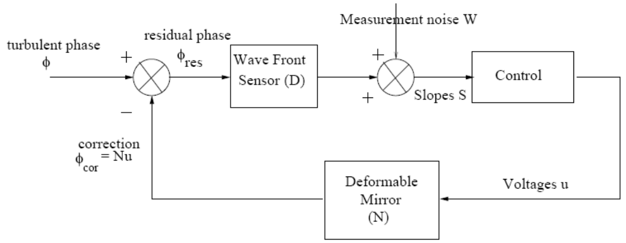
- **Optimal** reconstruction on very large space (many modes)
  & Projection of reconstructed phase onto DM space to obtain control voltages

Advantages:

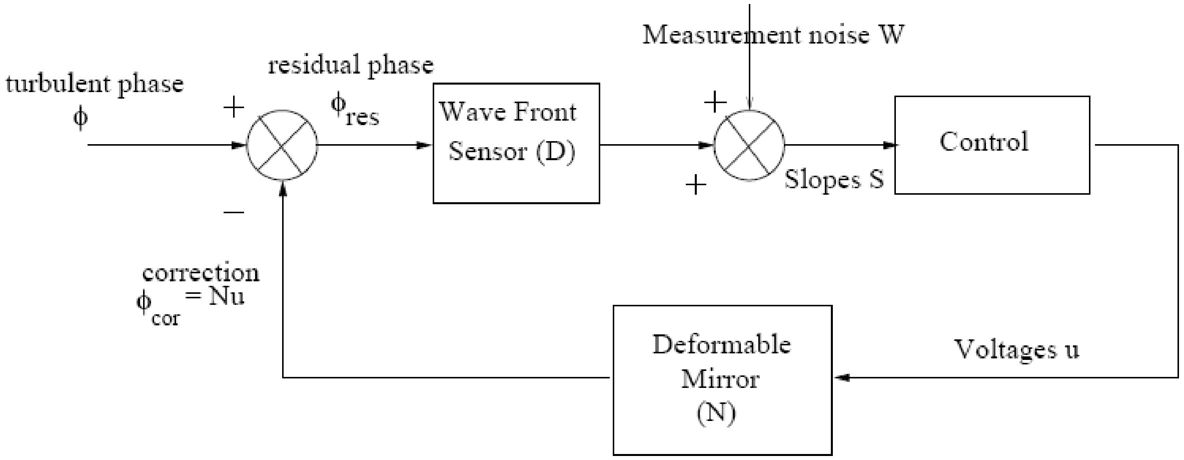
Optimal performance (good representation of turbulent and DM space…)

But requires to explicit models for turb. and noise statistics and for AO components

+ some specificities
of closed loop operation



AO Servo-Loop: standard control



Discrete time control:
Sampling time Ts
Sampling freq Fs=1/Ts

Correction phase: $\phi_{corr}(\mathrm{r}) = \sum_i \mathrm{u}_i \; N_i(\mathrm{r})$

- ❑ **Voltage increments reconstructed from the residual slope at time n:**

  $$\delta \mathrm{u}_n = R_{com} \, s_n = R_{com} \left[D \, \phi_{res \; n-d} + \mathrm{w} \right]$$

- ❑ **Integrator control law:**

  ☞ **Integrator :** $\mathrm{u}_n = \mathrm{u}_{n-1} + g \, \delta \mathrm{u}_n = \mathrm{u}_{n-1} + g \, R_{com} \, s_n$

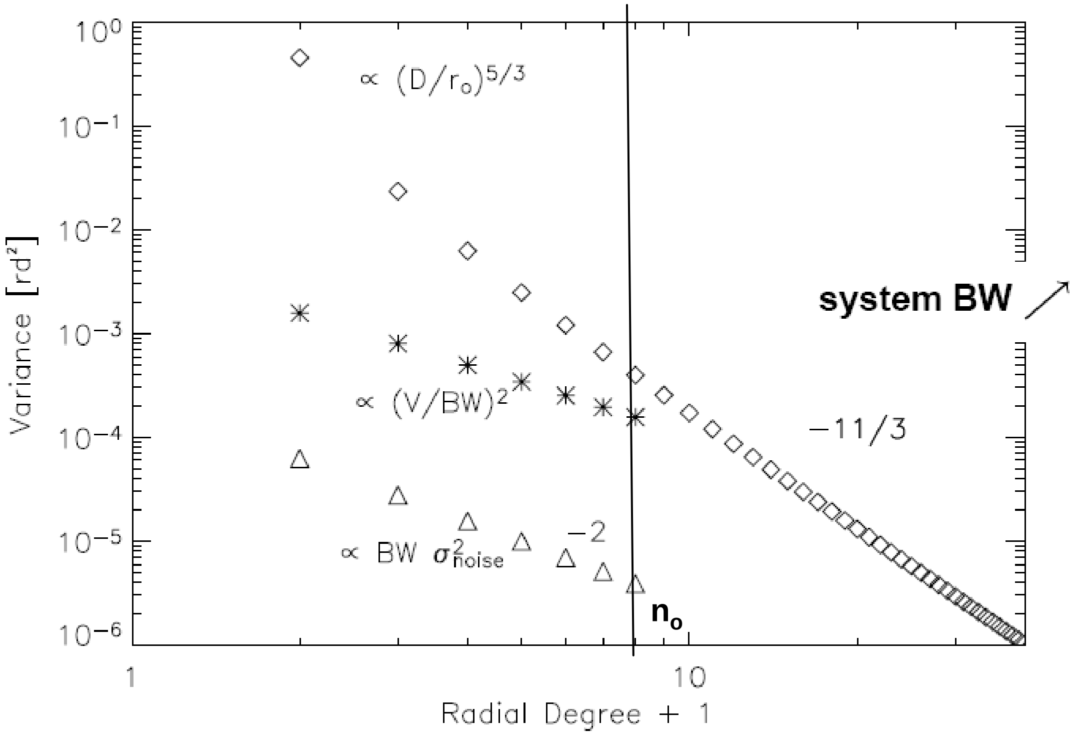
g is the loop gain ; it is related to system temporal BandWidth BW

system BW ↗ when loop gain g ↗

AO Global Error Budget

$$\sigma^2_{\phi\ res\ tot} = \sigma^2_{\phi\ tempo} + \sigma^2_{\phi\ noise} + \boxed{\sigma^2_{\phi\ high\ orders}}$$

So-called
Fitting Error



BW: system bandwidth
V: mean wind speed

system BW ↗ when loop gain g ↗

Fitting Error for no corrected radial orders:

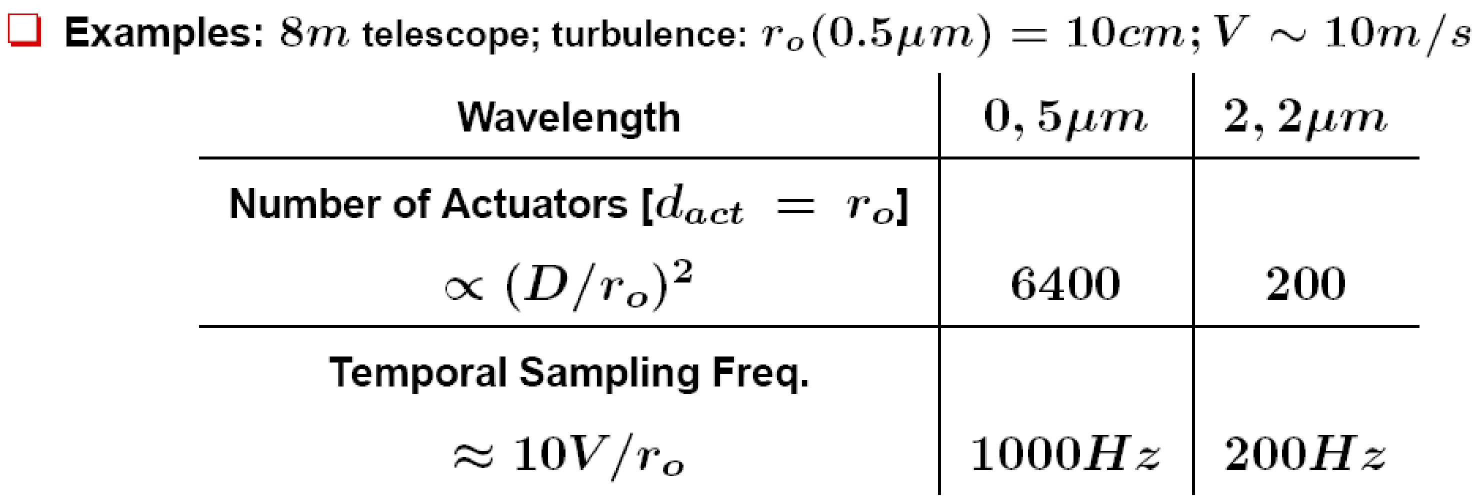
$$\sigma^2_\phi(n > n_o) \approx 0.46\,(n_o + 1)^{-5/3}\,(D/r_o)^{5/3}$$

AO Key Design Parameters

System design based on:

- observation conditions: r_o, V, star brightness, imaging wavelength
- performance requirements

Leads to choice of sampling frequency Fs, number of actuators…



☐ **Examples:** $8m$ telescope; turbulence: $r_o(0.5\mu m) = 10cm; V \sim 10m/s$

Wavelength	$0,5\mu m$	$2,2\mu m$
Number of Actuators [$d_{act} = r_o$] $\propto (D/r_o)^2$	6400	200
Temporal Sampling Freq. $\approx 10V/r_o$	$1000Hz$	$200Hz$

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From Very Large (VLTs) to Extremely Large (ELTs) Telescopes

Yesterday VLT

Diameter = 8m
Cerro Paranal - Chile

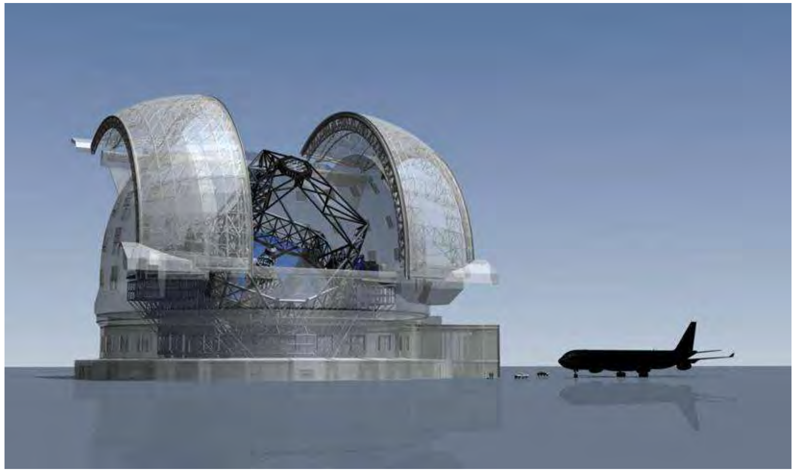
Today VLT updates:
DMs, Lasers, more fancy AO

First Light of the VLT Laser Guide Star

ESO PR Photo 07a/06 (23 February 2006)

© ESO

Tomorrow (~2020) E-ELT

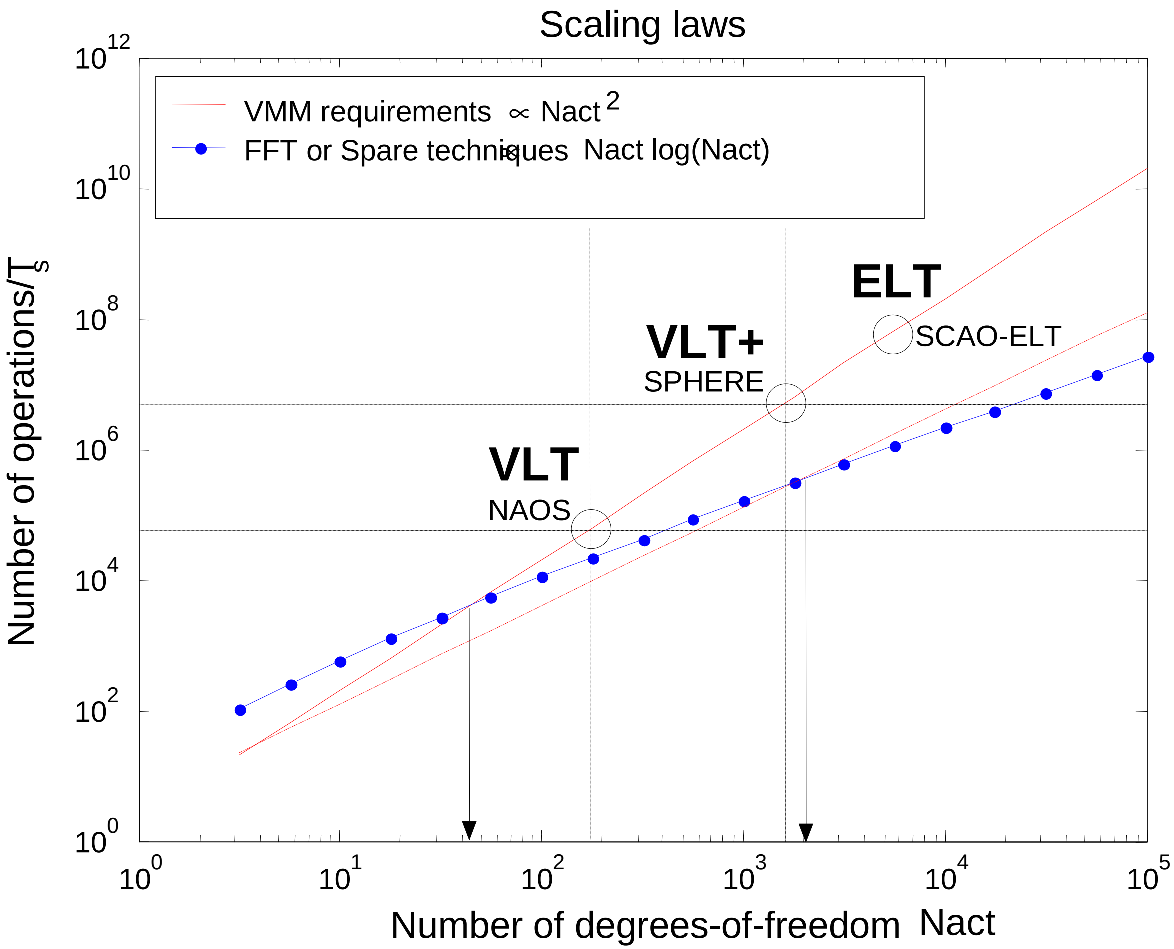


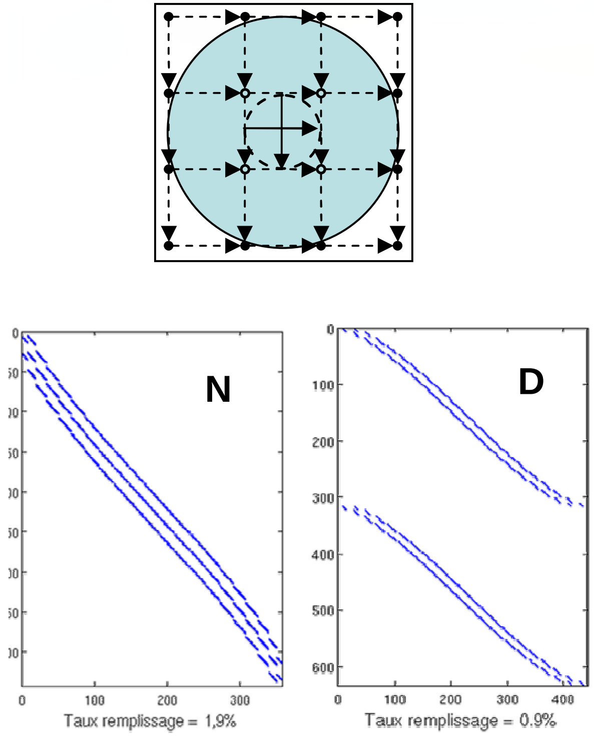
Diameter = 42m
Cerro Armazones - Chile

- **Large DM (2.5m)**
- **Large degrees of freedom:**
  Nb of actuators ~ 7000
- **Very challenging AO with**
  multi-WFS on Laser and Natural Stars

ESO = European Southern Observatory

AO Control Complexity: rather zonal basis for ELT?





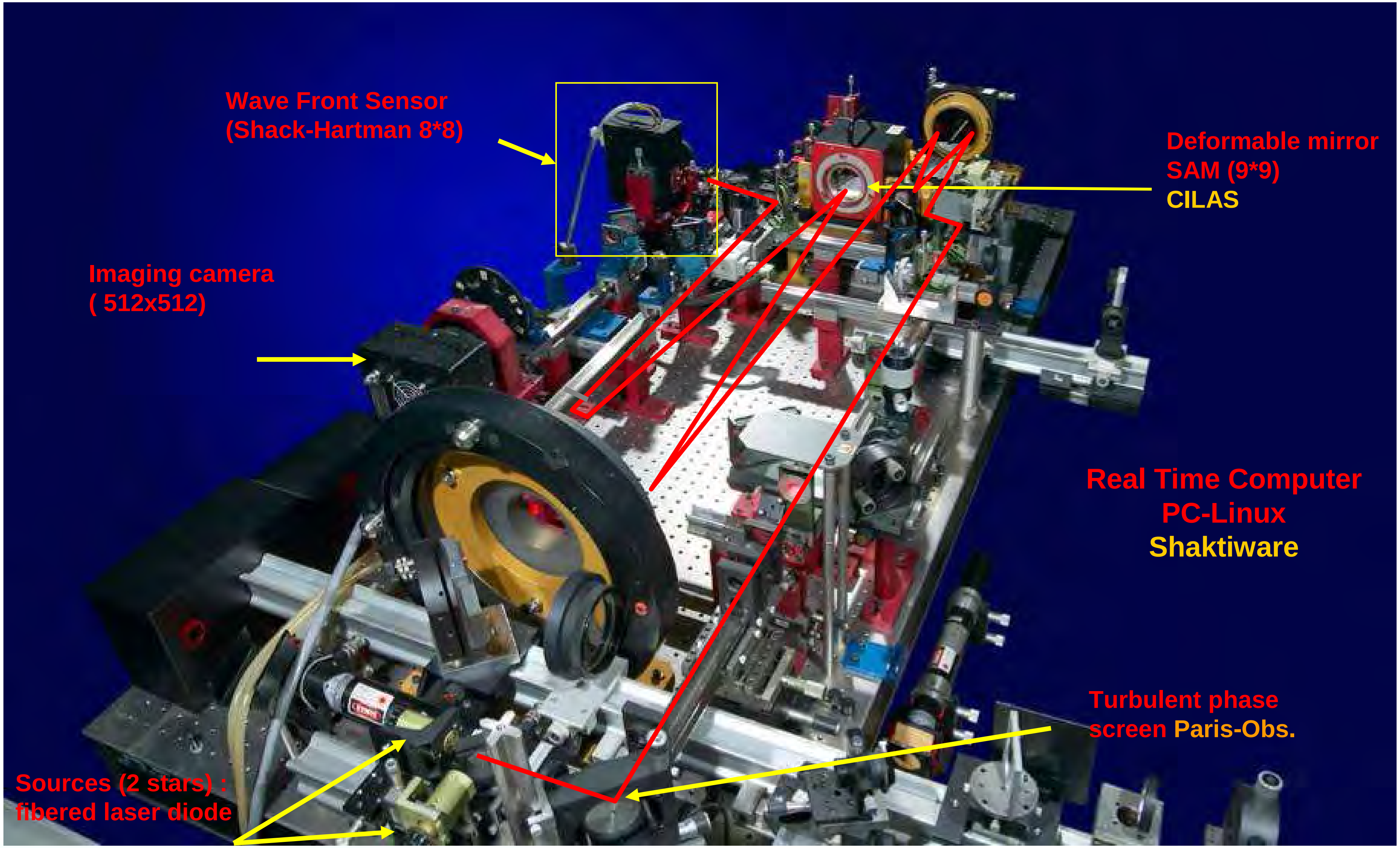
Computation complexity proportional to (Nact)² that is to D^4
Factor 625 when going from 8 to 40m telescopes!!

Computing efficient techniques based on FFT or sparse matrices implies zonal basis

Outline

- ☐ Atmospheric Turbulence
- ☐ Deformable Mirror
- ☐ Wave-Front Sensing
- ☐ Reconstruction and Control
- ☐ From Very to Extremely Large Telescopes
- ☐ Illustrations :
  - ☐ Adaptive Optics Systems and Images
  - ☐ The European Extremely Large Telescope

BOA Adaptive Optics Bench @ Onera


Wave Front Sensor
(Shack-Hartman 8*8)

Deformable mirror
SAM (9*9)
CILAS

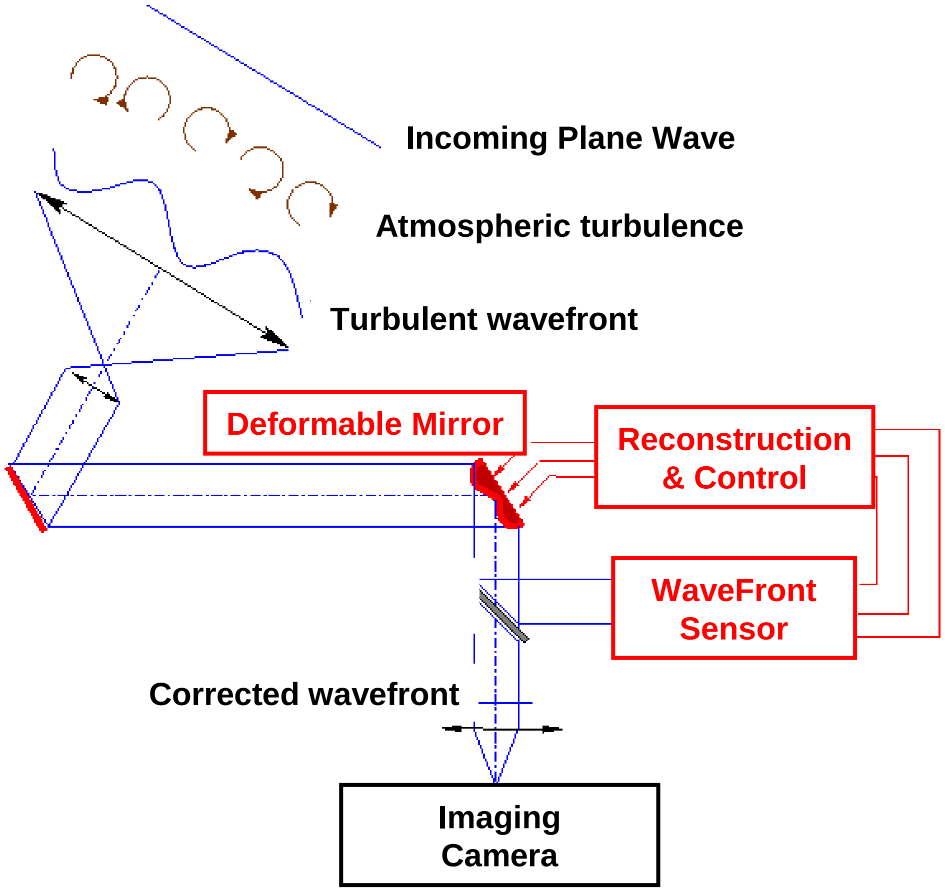
Imaging camera
(512x512)

Real Time Computer
PC-Linux
Shaktiware

Turbulent phase
screen Paris-Obs.

Sources (2 stars) :
fibered laser diode

Closing the Loop in Adaptive Optics



Incoming Plane Wave

Atmospheric turbulence

Turbulent wavefront

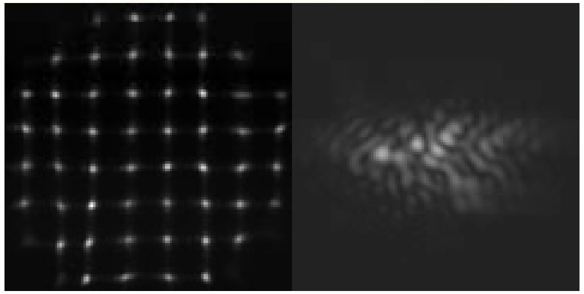
Deformable Mirror

Reconstruction & Control

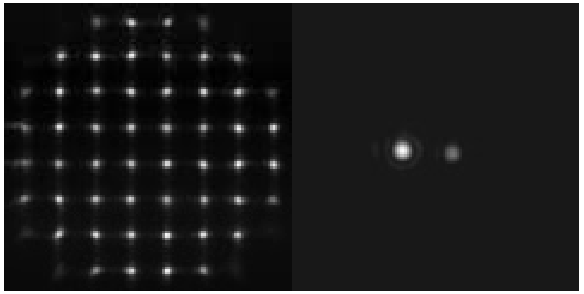
WaveFront Sensor

Corrected wavefront

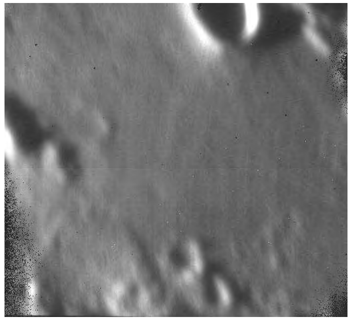
Imaging Camera



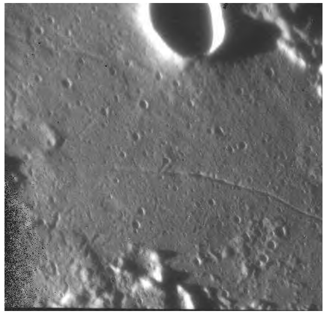
Open Loop



Closed Loop

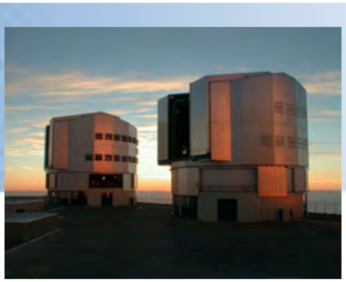


Uncorrected



Corrected

NAOS-CONICA since 2001 on the VLT

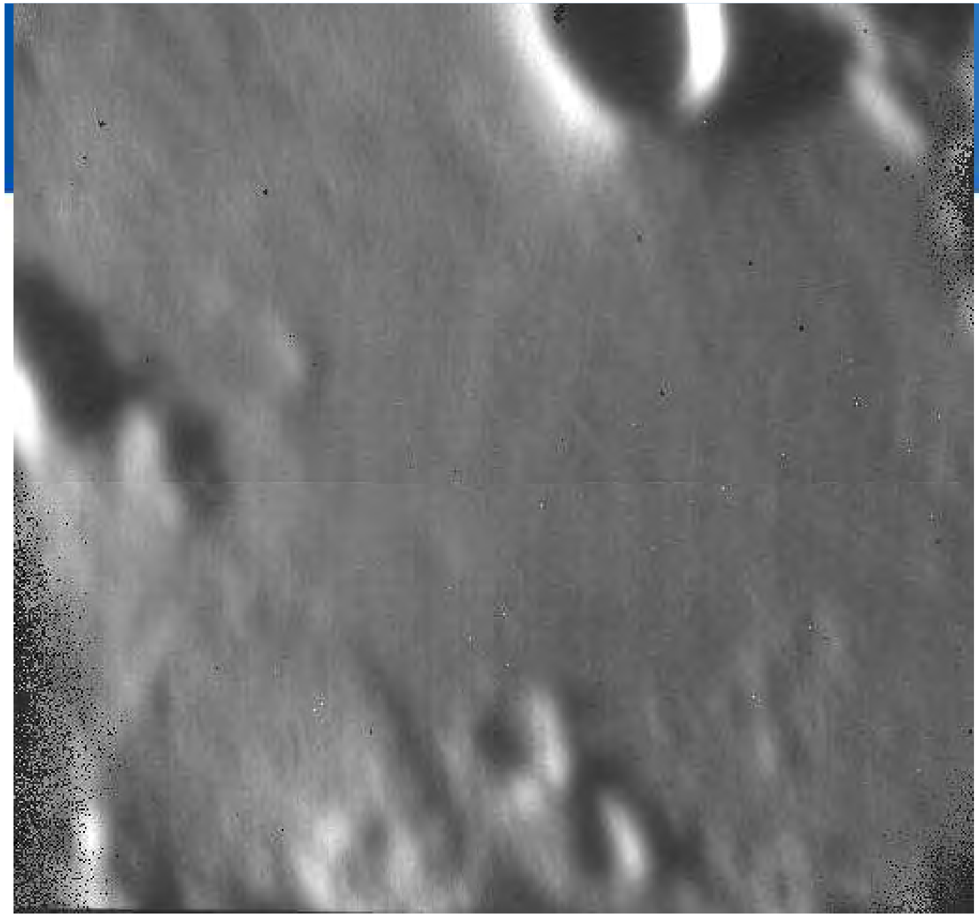


NAOS : AO system = 15x15 actuators ; 500 Hz sampling frequency
developed by: ONERA, Grenoble and Paris Observatories under ESO contract

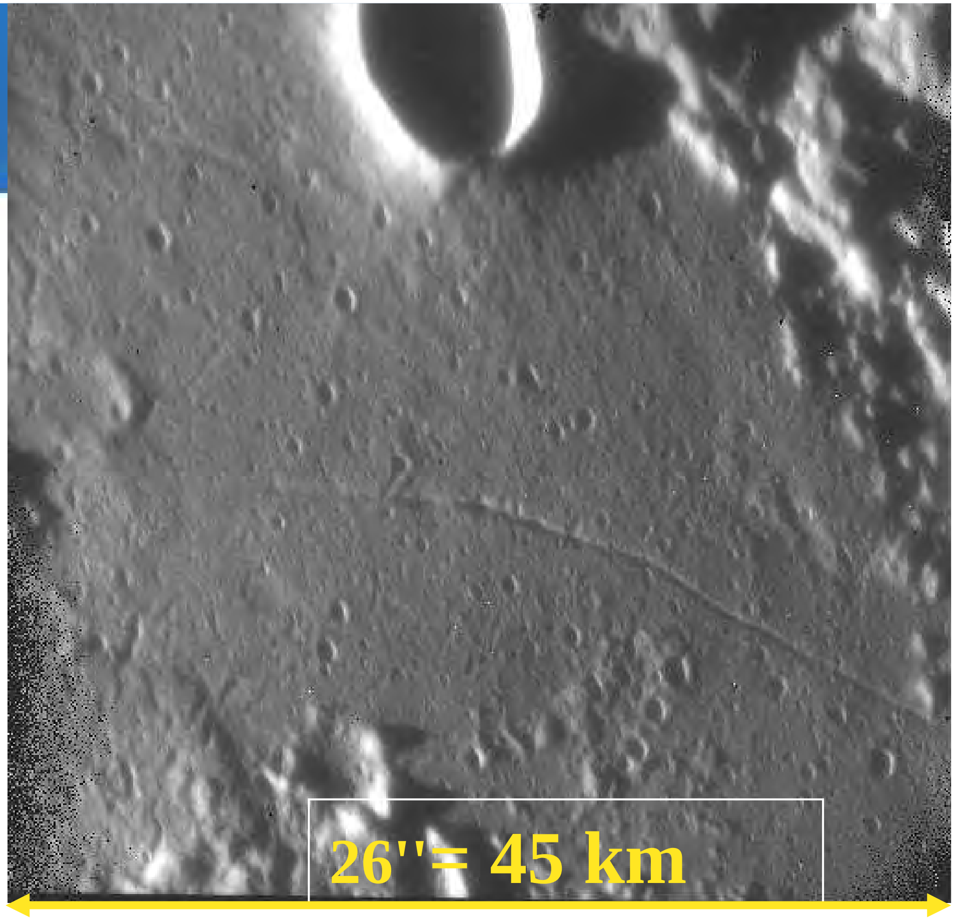
CONICA : near IR camera developped by Max Planck Institute

Many publications planetology, galactic, extra-galactic

Moon Observation with NAOS CONICA @ VLT



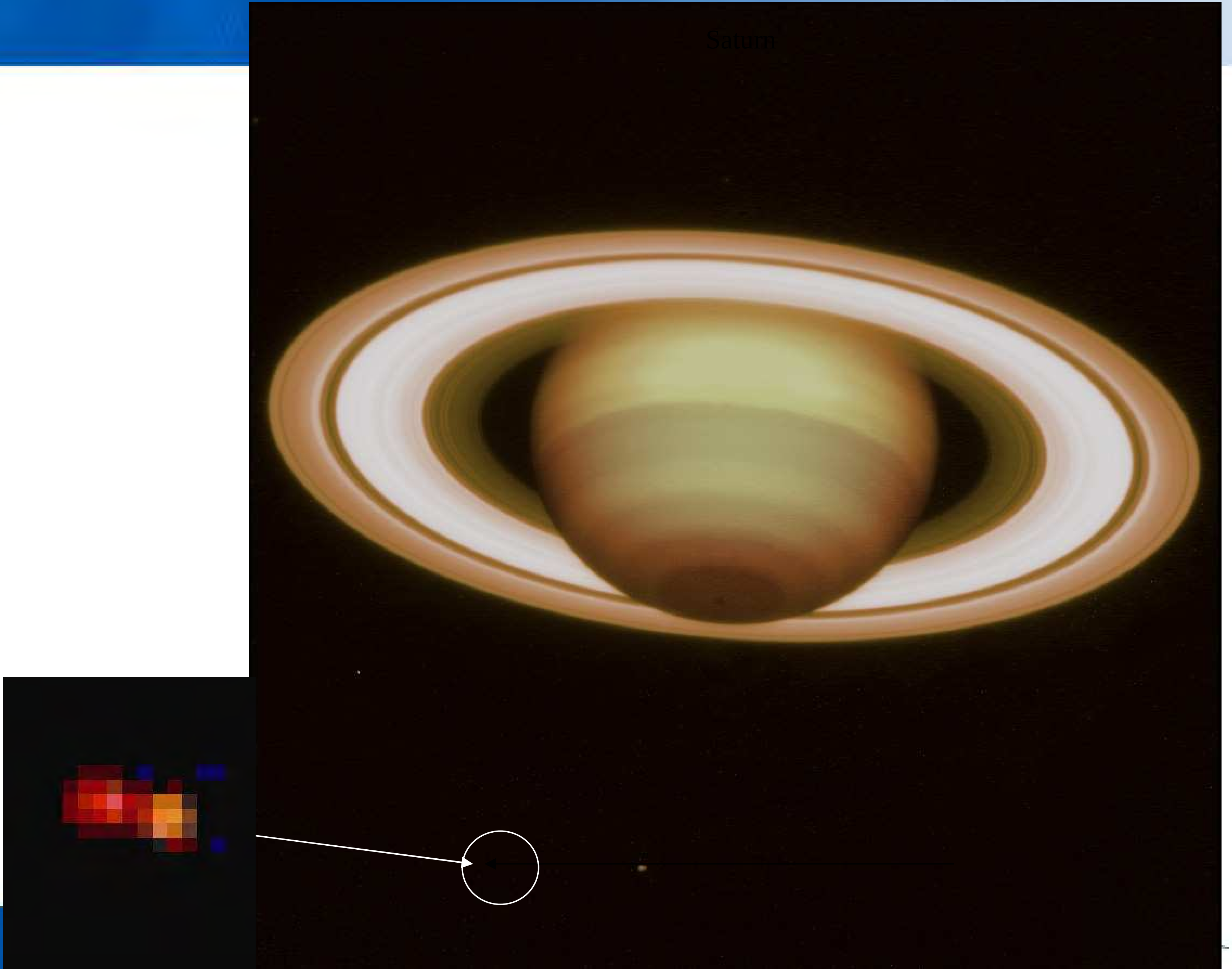
Open loop
wavelength = 2.3 microns



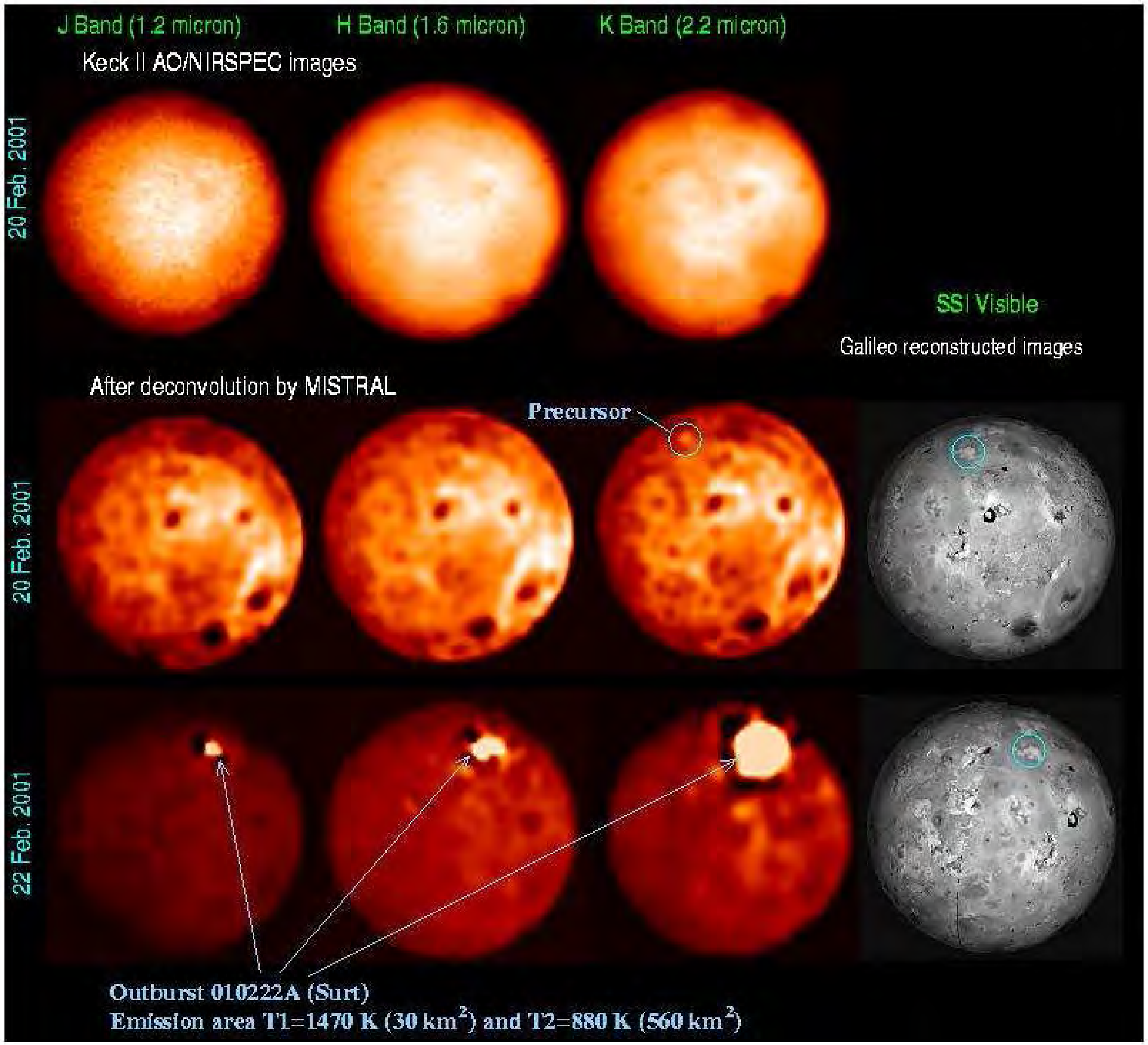
Closed loop
wavelength = 2.3 microns

WF sensing on bright lunar peak
20" south of FoV center (3" to 4'' diam)

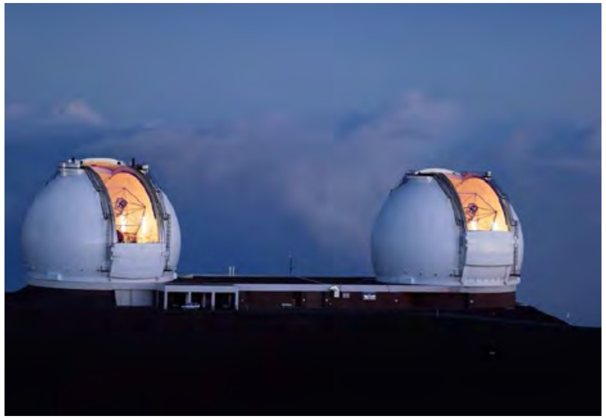
Saturn Image with NAOS CONICA @ VLT



Observation of Io (Jupiter satellite) with AO@Keck



Outburst 010222A (Surt)
Emission area T1=1470 K (30 km²) and T2=880 K (560 km²)



Exoplanets : direct detection with AO?

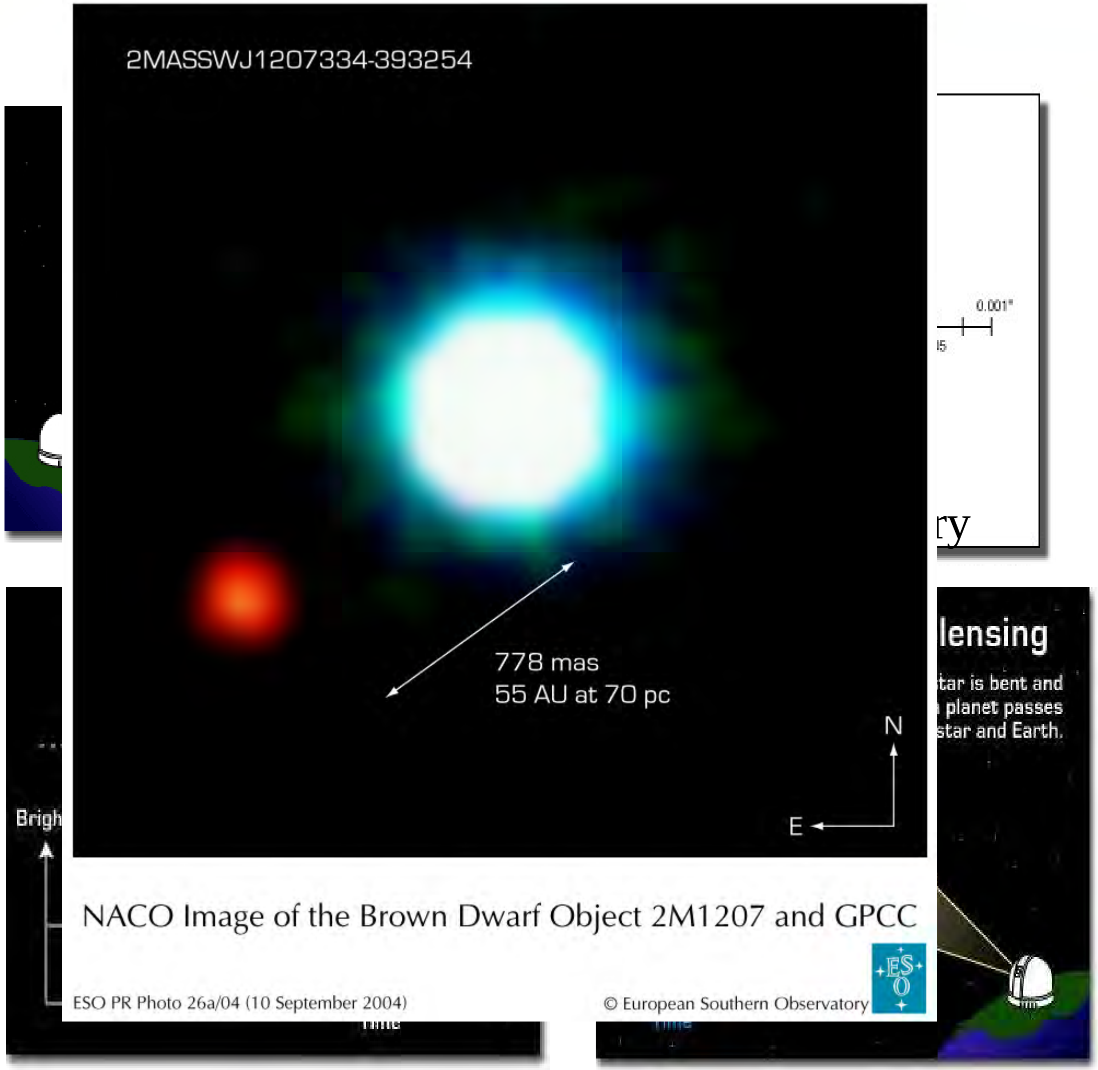
more than 100 exo-planets discovered up to now

Only through indirect methods

no exo-planet photons detected!!

Except for One Direct Detection :

last year with VLT-NAOS

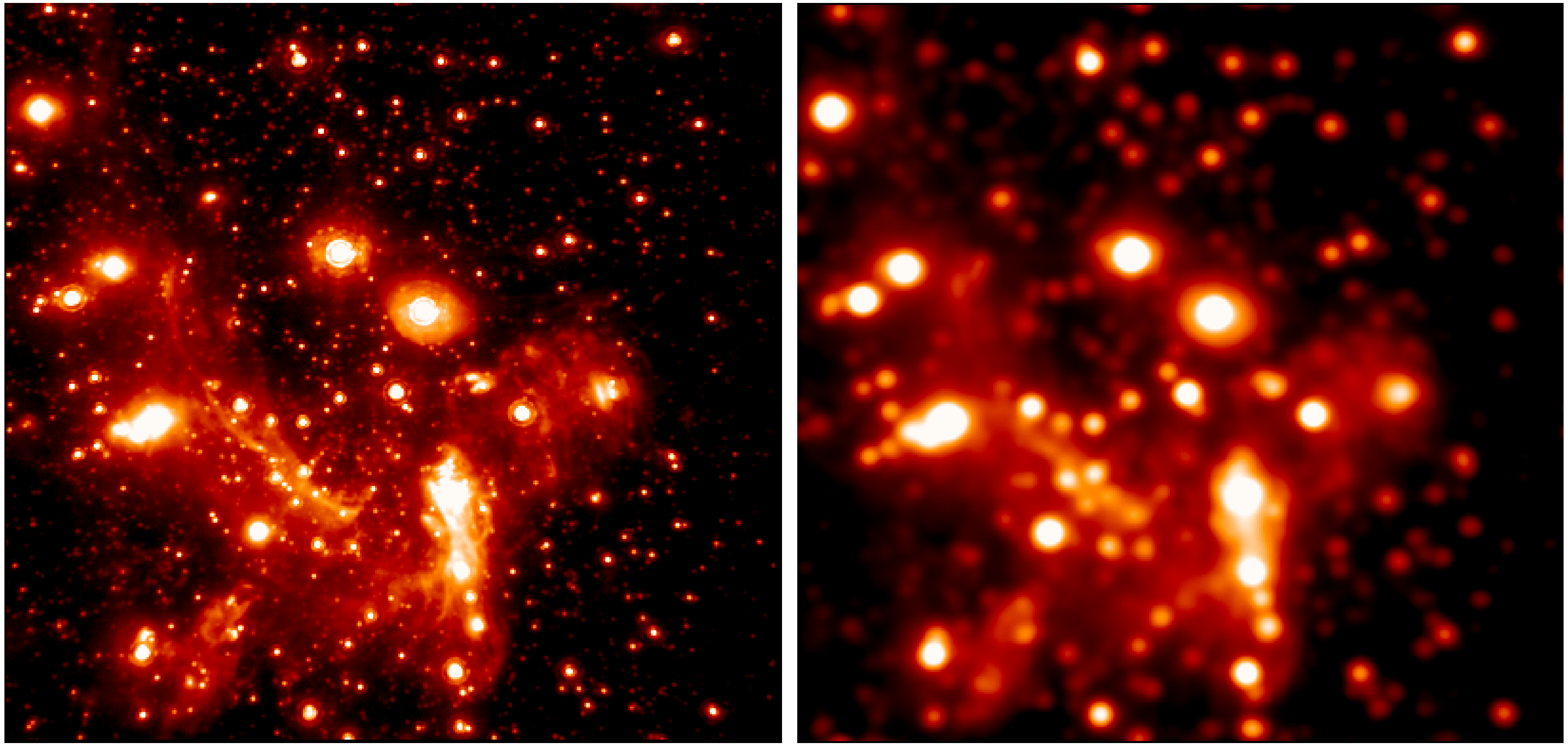


NACO Image of the Brown Dwarf Object 2M1207 and GPCC

ESO PR Photo 26a/04 (10 September 2004)

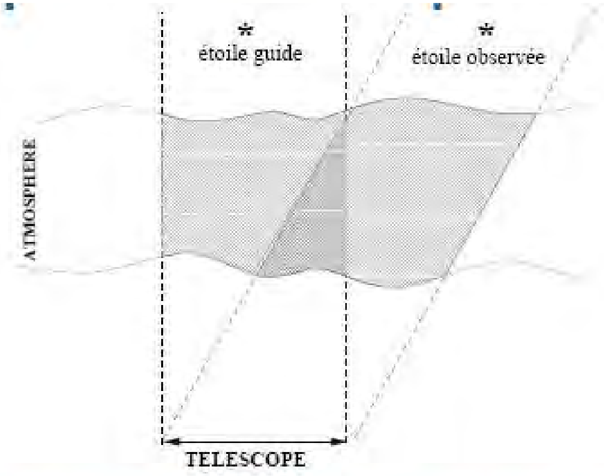
© European Southern Observatory

Galactic Center with NAOS CONICA @ VLT



Courtesy Y. Clenet Observatoire de Meudon

Adaptive Optics & Anisoplanatism Effect

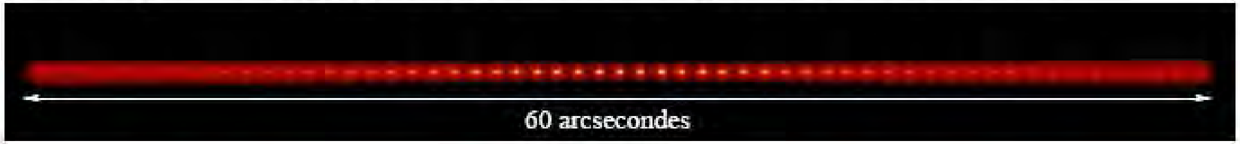


Anisoplanatism= evolution of the turbulent phase in the field of view :

turbulence in *volume*

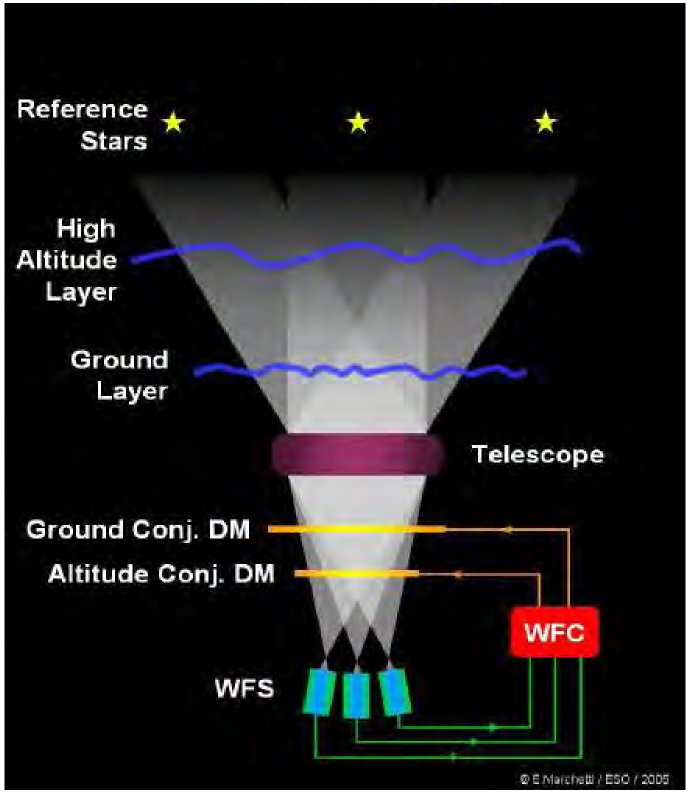
Classical Adaptive Optics:

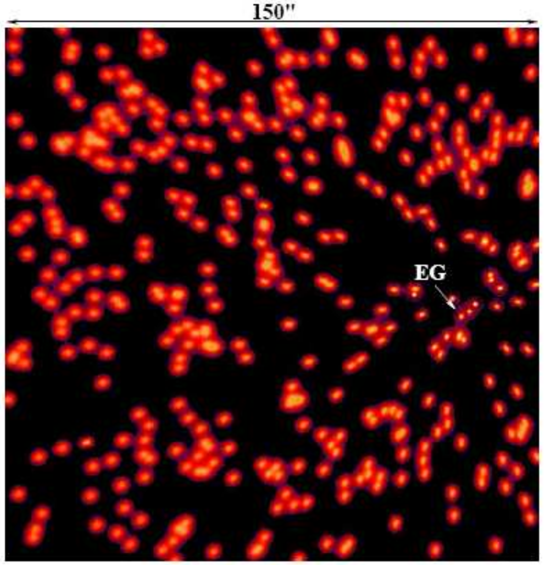
- ☐ 1 guide star for WFS, 1 object of interest
- ☐ 1 DM in pupil $\longrightarrow \phi_{corr}$ independent of position in the field



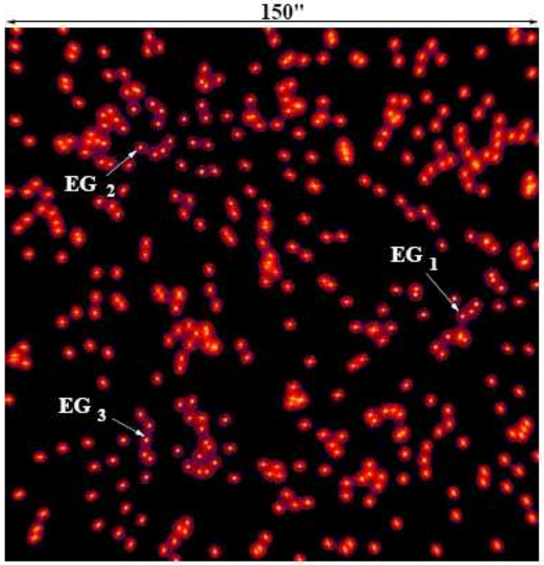
Correction quality degrades away from the guide star

MultiConjugate Adaptive Optics (MCAO)





regular AO

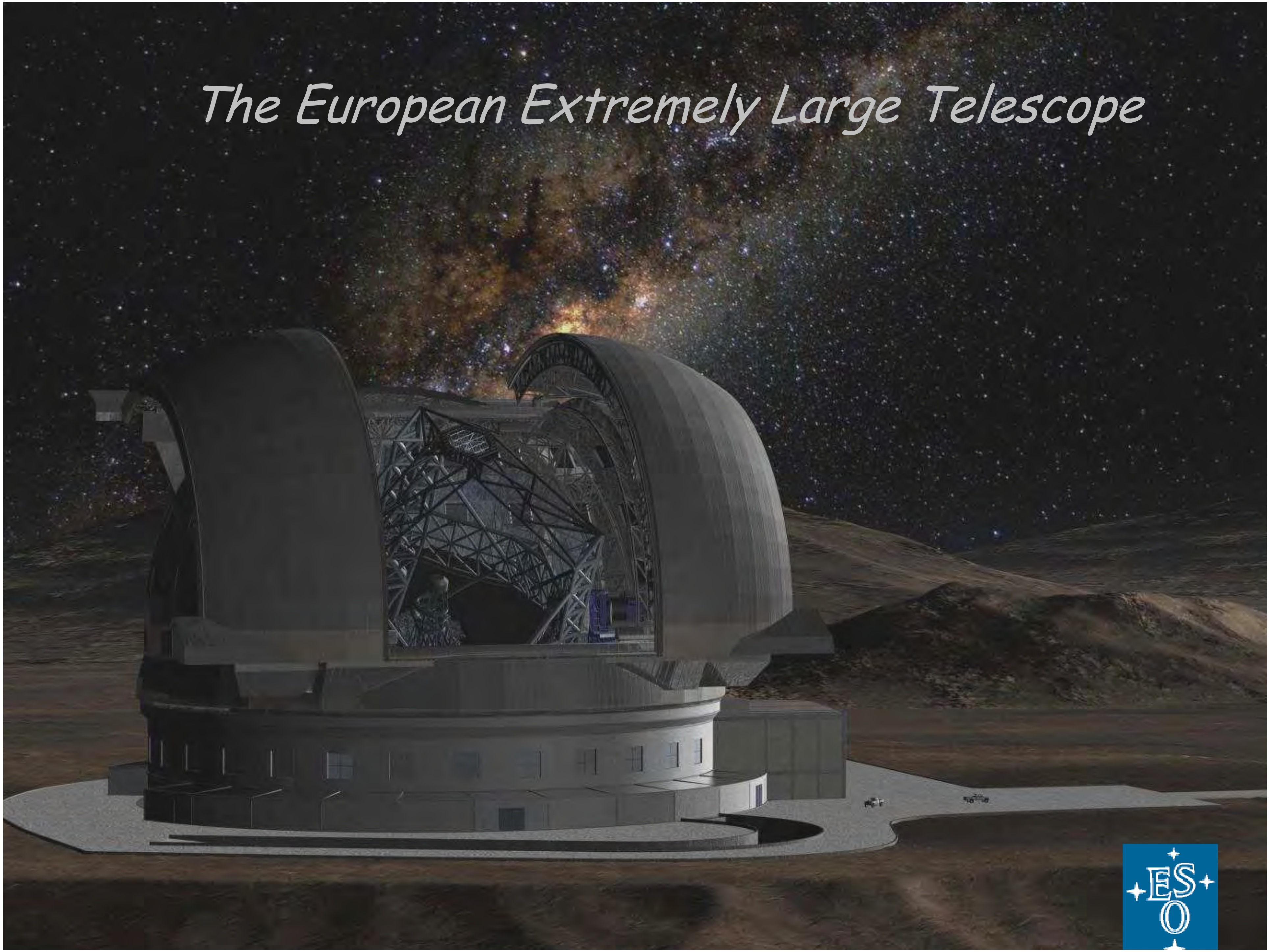


MCAO

Two major ingredients:

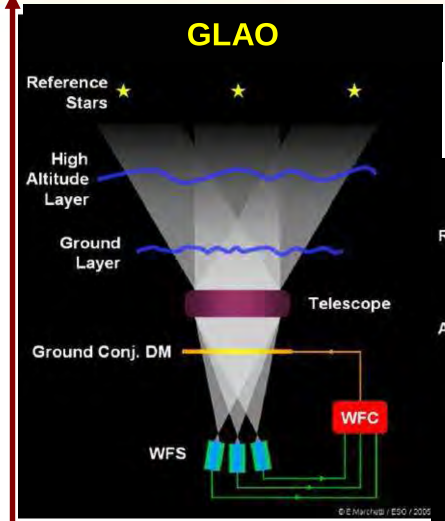
- Measurement of turbulence volume via multi-WFS
  (similar to perspective views in 3D imaging, to tomography in medical imaging…)
- Correction of volume with several DMs

The European Extremely Large Telescope

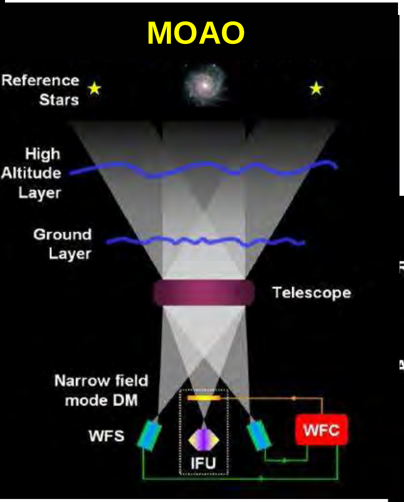


Zoology of E-ELT Wide Field Adaptive Optics

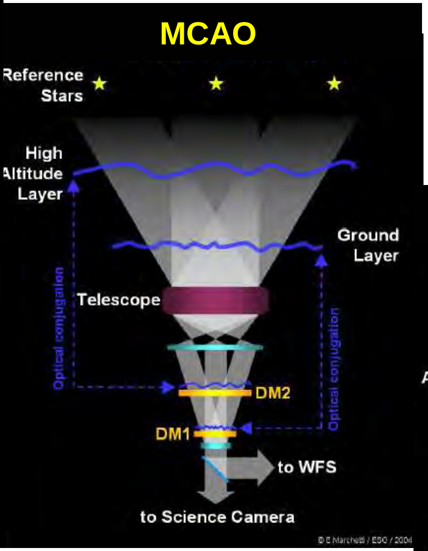
Corrected FoV



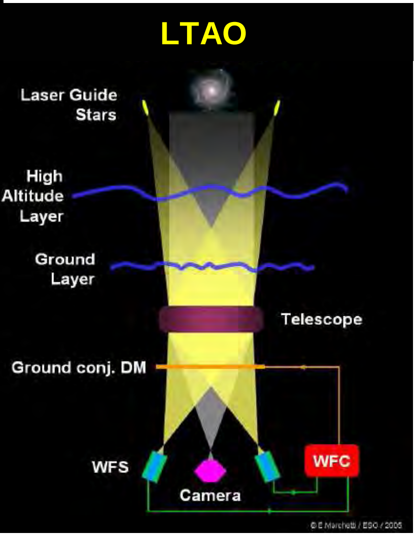
Seing enhancement
In 10x10 arcmin^2



EE > 30% in few arcsec^2
Multiplexing ~ 40 objects
In 5x5 arcmin^2



SR ~ 30%
In few arcmin^2



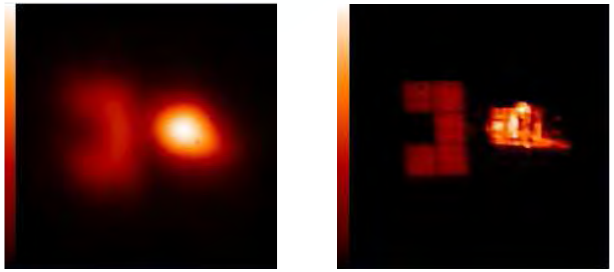
SR ~ 30 %
In few arcsec^2

Performance

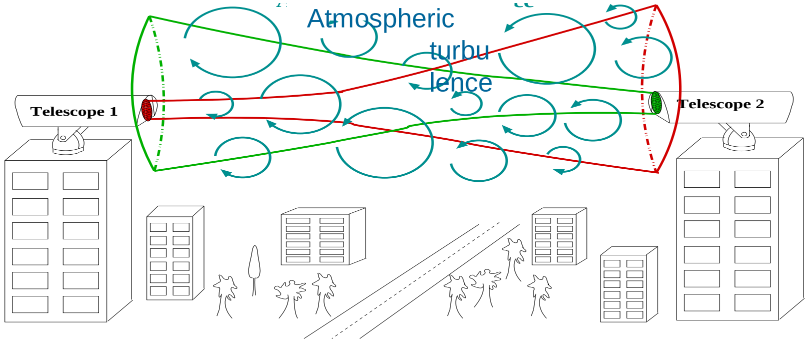
Advanced Tomography with Laser for AO systems

Adaptive Optics Applications: other applications

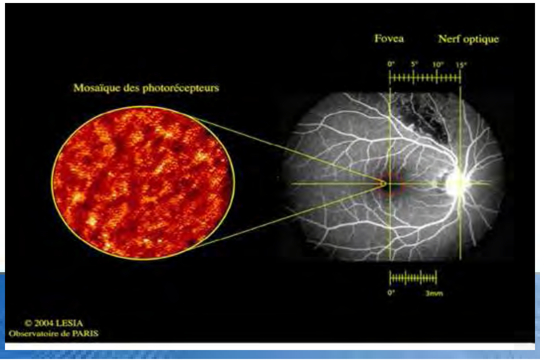
❑ Satellite Observation



❑ Free Space Optical Telecommunications



❑ Eye Retina Observation and Microscopy



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Questions VERY welcome

EXTREME thanks for your attention